To Serdar Sayan

A GENETIC ALGORITHM-BASED SEARCH FOR PARAMETRIC REFORM ALTERNATIVES FOR THE TURKISH PENSION SYSTEM: 2005-2060

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ABSTRACT

A GENETIC ALGORITHM-BASED SEARCH

FOR PARAMETRIC REFORM ALTERNATIVES

FOR THE TURKISH PENSION SYSTEM: 2005-2060

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In this thesis, we search for parametric reform alternatives so as to achieve a balance in the long term accumulated difference between pension expenditures and revenues of the SSI, the largest pension fund in Turkey, from 2005 to 2060. The projected worker-retiree composition of the pension system is allowed to change along with proposed policy changes in contribution and replacement rates. These changes in population structure are modeled by estimated work and pension income elasticities of labor supply values so that the resulting changes in the incomes of the worker could now affect the retirement decision. Possible policy alternatives are then found by a genetic algorithm developed for this purpose. Finally, the results obtained from the program are compared with previous and current reform proposals.

Keywords: Pension Reform, Genetic Algorithms

ÖZET

TÜRK EMEKLİLİK SİSTEMİ İÇİN PARAMETRİK REFORM

ALTERNATİFLERİNİN GENETİK ALGORİTMA YARDIMIYLA SAPTANMASI:

2005-2060

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Bu tezde, Türk emeklilik sisteminin 2005-2060 yılları arasındaki uzun vadeli gelir gider dengesini sağlayabilecek parametrik reform alternatifleri aranmaktadır. Çalışmada, öncekilerden farklı olarak SSK'ya tabi nüfusun uzun dönem tahmini çalışan-emekli bileşiminde bağlama ve prim oranlarında yapılacak ayarlamaları takiben gözlenecek değişiklikler gözönüne alınmaktadır. Bu değişiklikler, çalışan ve emekli maaşlarındaki hareketlerin işgücü yapısı üzerindeki etkilerini tahmin etmeyi kolaylaştıran, esneklik tahminleri kullanılarak modellenmektedir. Bu çerçevede muhtemel politika alternatifleri bu amaçla geliştirilen bir genetik algoritma yardımıyla saptanmaktadır. Elde edilen sonuçlar şu anda gündemde olan ya da yakın geçmişte uygulanmış reform alternatifleriyle karşılaştırılmaktadır.

Anahtar Kelimeler: Emeklilik Reformu, Genetik Algoritmalar

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CHAPTER 1

INTRODUCTION

The reason of existence for any government is to provide its citizens with some basic services such as security, justice, and education. In this framework, a social security system plays an important role by letting elderly citizens maintain at least a minimum living standard after they become unable to pay for their consumption with their labor.

A common approach to help retired workers pay for their own consumption is to operate a pension system which collects premiums from the current worker's paychecks to finance the elderly workers through a distribution system. The system is called Pay-As-You-Go (PAYG) to accentuate the re-distribution aspect.

The PAYG systems were first established in the second half of the 19th century to fund the current retirees with contributions out of current worker's income while promising the current workers the same treatment when they have reached their old age. The high ratio of workers to retirees at the time of their establishment enabled PAYG systems to run surpluses for several decades, despite generous retirement benefits they provided. In Europe, the baby-boomers of the 1940s and 1950s helped maintain these high worker retiree ratios, keeping the PAYG systems running without incurring deficits. Yet, increasing life-expectancies caused the number of retirees to increase and decreasing fertility rates slowed down the growth in the number of workers in many countries leading to pensions crises. It became obvious in many OECD countries, for example, that the fast-graying population cannot be funded by the current workers without initiating serious policy changes (Kenc and Sayan 2001a).

In one of the early contributions recognizing the problem, Auerbach et. al. (1989) studied four OECD countries using an OLG model and warned against the dangers of declining worker to retiree ratios ahead. Likewise, Chand and Jaeger (1996) noted the importance of the implications of aging in a society from the point of view of pension systems, and discussed how balances could be controlled by changes in the fundamental parameters of the system: the rate of contributions out of workers' wages (contribution rate), the rate of work time average of wages replaced by pension income (replacement rate), and the minimum retirement age. Chand and Jaeger (1996) concluded by pointing to the need to create a fully-funded, defined contribution scheme within or outside the existing PAYG systems, since the PAYG system balances would continue to deteriorate unless necessary steps are taken to ensure a smooth transition to accommodate the requirements of the demographic developments ahead to remedy the system.

The establishment of PAYG pension funds in Turkey followed the same natural path in the aftermath of World War 2 through the creation of three different pension funds: Emekli Sandığı (ES), Sosyal Sigortalar Kurumu (SSK), and Esnaf ve Sanatkarlar ve Diğer Bağımsız Çalışanlar Sigorta Kurumu (BAĞ-KUR), for three different types of workers: public sector employees, private sector employees, and self-employed craftsmen and artisans, respectively. The ratio of workers to retirees remained high in the following decades until the 1980s. In fact, this ratio was expected to stay high enough not to cause concern at least until the 2020s because of the high fertility rates in Turkey. Thus, unlike its counterparts in other OECD countries the Turkish pension system could be run without the need for drastic changes in pension parameters. Nevertheless, pension balances in Turkey quickly deteriorated starting from the 1990s due to the shortsighted and populist interventions by the policy makers (TUSIAD, 2004).

A political decision in 1992 aimed to reap the benefits of popular support lowered the minimum retirement age to as low as 38 in women and 43 in men who have paid their premiums for 20 years to earn their retirement. This implied that anyone contributing to the system for 20 years could continue to receive retirement benefits for the next 35 years (MLSS/SSI, 2004). The expected effect for the government was increased popularity among the working people by retiring them from the workforce earlier than usual and transferring their old jobs to younger workers to reduce unemployment, and hence to gain more popularity. Furthermore, the amount of pension income provided to the retirees often exceeded their active working wages, increasing people's incentive to retire early. The young retirees often continued to work in their old jobs after retirement thereby avoiding premium payments and complementing their wages with pension income. As a result, the expected employment generation effects were not observed (MLSS/SSI, 2004; Sayan, 2005). Combined with the mismanagement of accumulated funds during the early decades of the pension system, the lack of political will to stop leakages due to the number of unregistered workers, and the reluctance to punish the employers who did not pay their contribution rates on time, the pension system began to report considerable deficits after the 1990s (Sayan, 2005).

The government's first attempt to correct this problem (via the social security reform of 1999) involved increasing the minimum retirement age to 58 for women and 60 for men after a transition period of 10 years. However, the non-linear age increasing scheme was soon ruled out by the Constitutional Court on the grounds of violating social justice. The Court required a smoother transition in increasing retirement ages and the government had to extend the transition period to 20 years while decreasing the minimum retirement age to 56 for women and 58 for men.

The following figure taken from MLSS/SSI (2004) shows the transfers made by the Treasury to social security institutions in terms of the ratio of transfers to the nation's GDP.

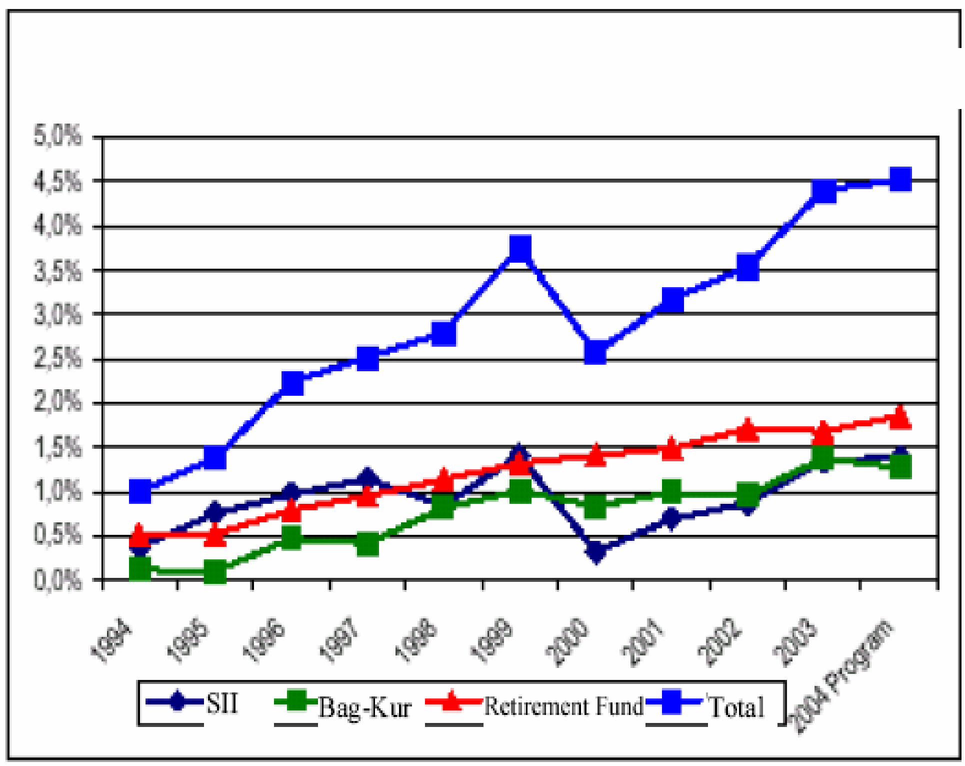

Figure 1: Transfers to Social Security Institutions by the Treasury

The figure shows a steady increase in transfers to GDP ratio until 1999 when a major parametric reform was legislated. The decline in the SSI deficit indicates the temporary success of the 1999 act to rectify the situation. However, with the help of a ruling of the Turkish Constitutional Court, which found the 1999 act to be violating social justice, the steady increase in budgetary transfers to social security institutions could not be stopped and total social security deficit reached up to 4.5% of the GDP in 2003. The amount of transfers made to these institutions in 2004 was 5 percent of the GDP, or USD 15 billion. Furthermore, total value of transfers made to the social security system by the end of 2003 discounted at Turkish Treasury Bill interest rates translates to 345 billion New Turkish Liras, or 1.24 times the total consolidated government debt at the time (MLSS/SSI, 2004).

Having recognized the magnitude of the problem, various researchers also studied parametric reform alternatives and their consequences. Sayan and Kenc (1999) used an overlapping generations (OLG) general equilibrium model to study the effects of increasing retirement ages in Turkey. Sayan and Kiraci (2001a) and Sayan and Kiraci (2001b) investigated alternatives based on grid search techniques to find reasonable retirement age, contribution and replacement rate combinations to minimize the pension deficit over a model horizon between 1995 and 2060. These studies were significant as they were the first attempts to formally model balances of the Turkish pension system in a 66-year horizon, forward-looking framework to suggest sustainable parameter configurations. Sayan and Turhan-Sayan (2001) used a genetic algorithm approach to speed up the slow grid-search process to identify parametric reform alternatives over the 2000-2060 period. They, however, did not consider the effects of changes in contribution and replacement rates on choices between work and retirement decisions.

An OLG model was used in TUSIAD (2004) to study the general equilibrium effects of a transition from the PAYG system to the funded system. The study showed that such a transition could boost the GDP as much as two percent per year.

The approach to be followed in this thesis is based on Sayan and Turhan-Sayan (2001). Even though the genetic algorithm developed there is very efficient, the interaction between the number of workers and retirees and changes in pension parameters could be modeled more realistically to find more relevant policy choices for the Turkish pension system.

Furthermore, Sayan and Turhan-Sayan (2001) create a model of the SSI system where optimal values of the retirement age can be searched for all years between 2000 and 2060, while the contribution and replacement rates can only be changed once and for all during the whole model horizon. Modifying the model in such a way to let all parameters be changed as frequently as a year would give a wider range of alternatives to policy makers.

The purpose of this thesis is to extend the work of Sayan and Turhan-Sayan (2001) by allowing for the projected worker-retiree composition to change along with the changes in contribution and replacement rates according to the estimated elasticity values so that the resulting changes in the incomes of the worker could now affect the retirement decision. Coupled with the possibility of yearly changes in contribution and replacement rates towards their optimal values introduced to the model here, it will become possible to observe whether the results in Sayan and Turhan-Sayan (2001) can be improved upon.

The organization of the thesis is as follows. The next chapter provides a brief introduction to the genetic algorithms. Chapter 3 describes the model used, while the associated results are presented in Chapter 4. Finally, Chapter 5 concludes the discussion.

CHAPTER 2

GENETIC ALGORITHMS

A genetic algorithm (GA) is a set of instructions that try to maximize or minimize an objective function by mimicking the survival-of-the-fittest mechanism of the nature as described in the Darwinian theory of evolution. To illustrate the idea, let us consider reproduction of a species. A population of species creates a pool of offspring, each born carrying a combination of the hereditary content of both of its parents. The offspring is then released into the nature. An offspring with a high quality genetic inheritance can adapt quickly to the nature and has a higher chance of survival and reproduction than others with a lower quality genetic inheritance. The offspring with higher quality genetic material eventually reach the reproduction stage where they mate with other high-quality genetic material offspring to produce more offspring that are even better equipped to survive. Certain mutations along the way could further enhance the survival chances of upcoming parents and hence their offspring. Thus, the survival-of-the-fittest mechanism explains how genetic material improves to make the population better suited to their surroundings with the passage of each generation.

The use of this idea in solving an optimization problem was first suggested by John Holland in the early 1960s (Holland, 1992). To see how he has transformed the elements of the theory of natural selection as the building blocks of a computer algorithm, one must first introduce basic concepts of the natural selection process.

Genes are the most crucial element distinguishing an individual from others and the sole constructs that carry information about an individual's inherited characteristics. They are basically certain molecules (bases) lined up on a long molecule-chain called the DNA. In nature, a DNA strand contains four different bases adenine, thymine, guanine and cytosine (abbreviated A, T, G, and C respectively). The line-up of these bases contains instruction codes for every chemical process in the cell, defining the fitness of the cell in nature. Shortly, genes in biology represent a string that carries an individual's unique properties for nature's processing. Such a string of length *n*, where *n* is a natural number, contains n^4 different combinations of line-up and carries immense possibilities for biological diversity.

Even though combining four different symbols to construct a string is also possible in computer science, it is generally easier to construct a string made up 0's and 1's (called a binary string or a chromosome) due to the binary nature of computation theory. This is a very useful concept since the bit-strings formed to represent the genetic inheritance properties of the cell can now be easily evaluated by a fitness function. A binary bit string A is formally represented as $A \in \{0,1\}^n, n \in N$, e.g.

A 6 bit binary string $\rightarrow$ 0 1 0 0 1 0

Corresponding to sexual reproduction in nature is a biological term, cross-over. Crossing over is the creation of new genetic content from two parents by means of a random exchange of the corresponding parts of the DNA strands. Crossing over process promotes diversity in the nature and helps produce new generations capable of adapting to nature. This process has its counterpart in a GA which works through the exchange of the bits indexed before the cross-over point with each other while keeping the chromosome order. The process yields two new chromosomes. The cross-over operation can be formally described by letting $A, B \in \{0,1\}^n$ be the binary strings and $c \in \{1,...,n-1\}$ be the randomly chosen cross-over point. Now, let any binary string *A* be divided into two binary strings A_c^- and A_c^+ where A_c^- represents the binary string formed by the first c bits of *A*. Naturally, A_c^+ represents the rest of *A*. Then, the functional form becomes:

$$CrossOver(A, B, c) = (C, D), where, C = A_c^- B_c^+, D = B_c^- A_c^+$$

A 6 bit Crossing-Over Example where c=3:

A	1	0	1	|0	0	0	B	0	1	0	|1	0	1
C	1	0	1	|1	0	1	D	0	1	0	|0	0	0

The nature's role in this process is twofold. First of all, it decides on the offspring's genetic content quality. In a GA, this role is captured by assigning a fitness value to the bit-strings with the help of a fitness function. Formally, a fitness function can be represented as:

$$f(A) \rightarrow F, A \in \{0,1\}^n, F \in \Re$$

where A is a length *n* binary string and *F* is a real number. The function f is generally assumed to assign higher values to the fitter binary strings.

The second role of the nature is the selection of pairs for reproduction. Genetically fit individuals have a higher chance to survive. They may also have some additional characteristics that help them find mates more easily than others. Then, those positive qualities of that individual would have a higher chance of being passed on to the next generation. This is replicated in a GA by ranking the fitness of individuals and associating a reproduction criterion to that ranking. For instance, “The Roulette Wheel Parent Selection Technique” takes the fittest (say 10) chromosomes from the population and lets them reproduce according to their fitness values. This way, each chromosome gets to reproduce with a probability $P(R_k) = \frac{F_k}{\sum_{i=1}^{10} F_i}, k \in \{1,2,...,10\}$. In cases where some of the fitness values are negative, it is always possible to assign positive values to the fitness values by adding a positive constant to all of the fitness values. Since the value of the added constant affects the probability distributions greatly, the selection of this constant is an issue of algorithm design. If one does not want to assign probabilities, the same probability for reproduction can always be assumed for all, again say 10, fittest chromosomes.

However, in some cases the whole population might be involved in reproduction instead of the fittest, say, ten. This would decrease the best individuals’ rates of survival but increase the diversity in the population to create more potent candidates for reproduction. This is called soft selection and has been shown to outdo some hard selection algorithms in terms of more rapid rates of improvement in for some evolutionally stagnant populations. Galar (1989) shows that soft selection methods proved to be more efficient than hard ones in crossing Gaussian multimodal regions. This is a result of the adaptability that the least fit individuals might bring into the system.

Another important factor in natural selection process is mutations. Mutations are changes in the gene-content of the offspring independent of the crossing-over process. When applied to GA’s mutations are random one or possibly more bit changes in the chromosome. With some preassigned probability, a zero at a particular location in the bit-string of an offspring may be changed to one or a one may be changed to zero.

Based on the terminology introduced, the algorithm can be verbally described as follows. After binary strings and the fitness functions are designed, an initial population is created randomly. That population is then evaluated by the fitness function. At the next stage, the population creates a new generation by means of crossing-over and mutation. This population too is ranked according to the fitness value. If a preset fitness criterion is satisfied by any of the chromosomes, the algorithm stops. If not, a new population is initiated from the existing one.

A flowchart for the algorithm is given in the next page in Figure 2.

A simple optimization task can be designed to demonstrate the use of genetic algorithms. Consider a simple optimization problem such as a finding the nearest point to the zero of a cubic function defined over a given interval.

Assume that the algorithm searches for the point which makes the value of the curve $f(x) = (x-4)^3$ closest to zero. Naturally, the desired point for this curve is attained at $x = 4$ and the desired value is 0. Now, let the search be in the region $x \in [0, 7.5]$. This means that the chromosomes making up the population should be designed so that their range is $x \in [0, 7.5]$. Now, define a 4-bit chromosome in this range such that every chromosome represents a different point in the $x \in [0, 7.5]$ region to represent the whole population of possible solutions. Thus the solution is said to have a 4-bit depth in the desired solution domain. Furthermore, imposing a linear distribution of points in the domain might be desirable and convenient. Then, a function $g(A)$, which has a 4-bit binary string or equivalently a four-digit number in modulo 2 as domain, maps $A \in \{0,1\}^4$ to its

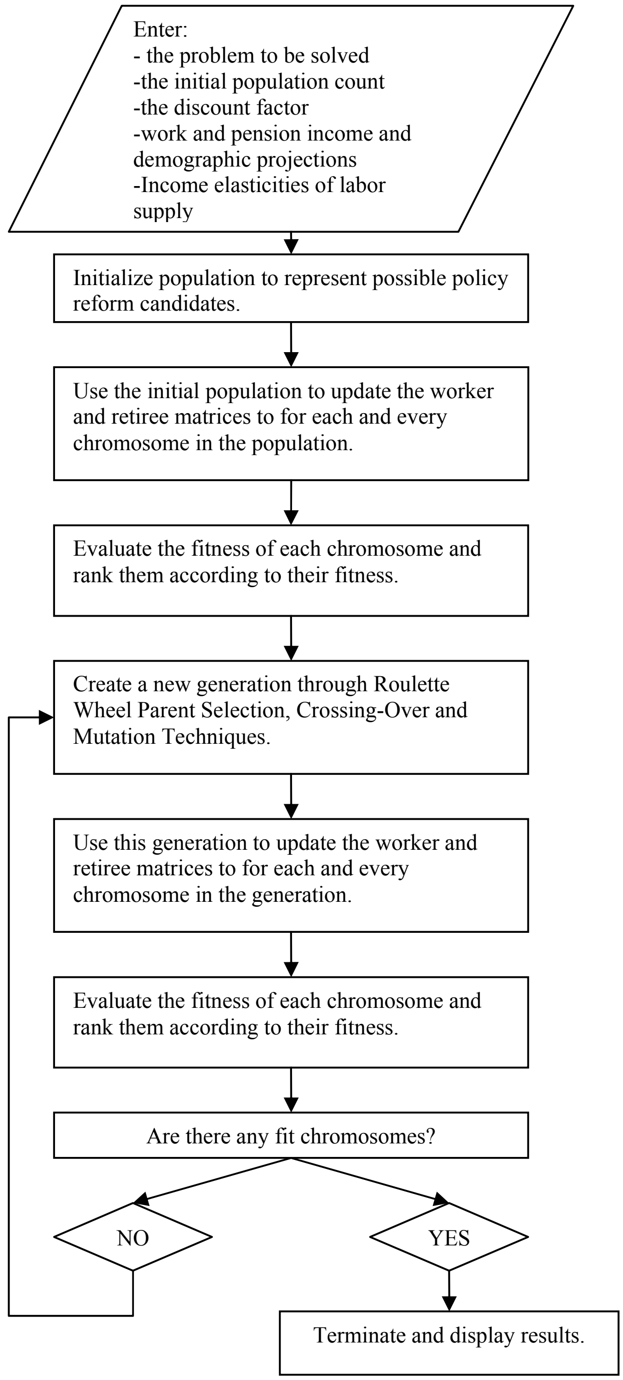

Figure 2: Flowchart for the Main Genetic Algorithm

relative range value, $x \in [0, 7.5]$. Here $g(A) = 0 + \frac{1}{2}\text{mod}_2(A)$, where 0 is the starting point of the fitness function's search range ($x \in [0, 7.5]$), $\frac{1}{2}$ is the step size between each element in the search range, and $\text{mod}_2(A)$ represents the possible values that any chromosome might take. Note that the minimal and maximal domain values for the fitness functions are achieved at $A = 0000$ and $A = 1111$, respectively.

$$g(0000) = 0 + \frac{\text{mod}_2(0000)}{2} = \frac{0 \cdot 2^0 + 0 \cdot 2^1 + 0 \cdot 2^2 + 0 \cdot 2^3}{2} = \frac{0}{2} = 0$$
$$g(1111) = 0 + \frac{\text{mod}_2(1111)}{2} = \frac{1 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3}{2} = \frac{15}{2} = 7.5$$

The fitness of each chromosome should increase in proportion with its closeness to zero. Assigning the fitness function below to the chromosomes fulfills this requirement.

$$F(A) = \frac{1}{|f(g(A))| + 2} = \frac{1}{|(g(A) - 4)^3| + 2} = \frac{1}{|(\frac{\text{mod}_2(A)}{2} - 4)^3| + 2}$$

where the constant 2 is added to the denominator of the function, $F(A)$, to avoid any possible zeros in the denominator.

Now, the population is initiated with four chromosomes chosen randomly from the whole population of $2^4 = 16$. Then, the respective reproduction probability for a four-chromosome initial population is

$$P(A_k) = \frac{F(A_k)}{\sum_{i=1}^{10} F(A_i)}, k \in \{1, 2, 3, 4\}$$

Let these initial chromosomes, their associated fitness and their reproduction probabilities be given as

$$A_1^1 = 1100 \longrightarrow g(A_1^1) = 6.0 \longrightarrow F(A_1^1) = 0.1000 \longrightarrow P(A_1^1) = 0.0808$$
$$A_2^1 = 0111 \longrightarrow g(A_2^1) = 3.5 \longrightarrow F(A_2^1) = 0.4760 \longrightarrow P(A_2^1) = 0.3804$$
$$A_3^1 = 1010 \longrightarrow g(A_3^1) = 5.0 \longrightarrow F(A_3^1) = 0.3333 \longrightarrow P(A_3^1) = 0.2694$$
$$A_4^1 = 0110 \longrightarrow g(A_4^1) = 3.0 \longrightarrow F(A_4^1) = 0.3333 \longrightarrow P(A_4^1) = 0.2694$$

As seen from the above results, the fittest chromosome is the 2nd one since it has the highest probability of passing on to the next generation. These probability values are then used to create the next generation by "The Roulette Wheel Parent Selection Technique". Assume that after the necessary random number generations, the following chromosomes were attached to each other in the following cross-over places.

$$A_1^2 = CrossOver(A_3^1, A_1^1, 1) = 1100$$
$$A_2^2 = CrossOver(A_4^1, A_2^1, 2) = 0111$$
$$A_3^2 = CrossOver(A_2^1, A_3^1, 1) = 0010$$
$$A_4^2 = CrossOver(A_3^1, A_4^1, 3) = 1010$$

The second generation's fitness values are reported below.

$$A_1^2 = 1100 \longrightarrow g(A_1^2) = 6.0 \longrightarrow F(A_1^2) = 0.1000 \longrightarrow P(A_1^2) = 0.0807$$
$$A_2^2 = 0111 \longrightarrow g(A_2^2) = 3.5 \longrightarrow F(A_2^2) = 0.4760 \longrightarrow P(A_2^2) = 0.3800$$
$$A_3^2 = 0010 \longrightarrow g(A_3^2) = 1.0 \longrightarrow F(A_3^2) = 0.0345 \longrightarrow P(A_3^2) = 0.2701$$
$$A_4^2 = 1010 \longrightarrow g(A_4^2) = 5.0 \longrightarrow F(A_4^2) = 0.3333 \longrightarrow P(A_4^2) = 0.2691$$

The third generation is reproduced by the same process.

$$A_1^3 = CrossOver(A_4^2, A_2^2, 1) = 1111$$
$$A_2^3 = CrossOver(A_2^2, A_4^2, 2) = 0110$$
$$A_3^3 = CrossOver(A_1^2, A_4^2, 3) = 1100$$
$$A_4^3 = CrossOver(A_2^2, A_1^2, 2) = 0100$$

The third generation's fitness values are also reported below.

$$A_1^3 = 1111 \longrightarrow g(A_1^3) = 7.5 \longrightarrow F(A_1^3) = 0.0223 \longrightarrow P(A_1^3) = 0.0233$$
$$A_2^3 = 0110 \longrightarrow g(A_2^3) = 3.0 \longrightarrow F(A_2^3) = 0.3333 \longrightarrow P(A_2^3) = 0.3488$$
$$A_3^3 = 1100 \longrightarrow g(A_3^3) = 6.0 \longrightarrow F(A_3^3) = 0.1000 \longrightarrow P(A_3^3) = 0.1046$$
$$A_4^3 = 0100 \longrightarrow g(A_4^3) = 4.0 \longrightarrow F(A_4^3) = 0.5000 \longrightarrow P(A_4^3) = 0.5232$$

The fourth generation is reproduced by the same process.

$$A_1^4 = CrossOver(A_4^3, A_3^3, 2) = 0100$$
$$A_2^4 = CrossOver(A_4^3, A_4^3, 3) = 0100$$
$$A_3^4 = CrossOver(A_3^3, A_2^3, 2) = 1110$$
$$A_4^4 = CrossOver(A_3^3, A_3^3, 2) = 1100$$

The corresponding fitness values and reproduction probabilities are:

$$A_1^4 = 0100 \longrightarrow g(A_1^4) = 4.0 \longrightarrow F(A_1^4) = 0.5000 \longrightarrow P(A_1^4) = 0.4407$$
$$A_2^4 = 0100 \longrightarrow g(A_2^4) = 4.0 \longrightarrow F(A_2^4) = 0.5000 \longrightarrow P(A_2^4) = 0.4407$$
$$A_3^4 = 1110 \longrightarrow g(A_3^4) = 7.0 \longrightarrow F(A_3^4) = 0.0345 \longrightarrow P(A_3^4) = 0.0304$$
$$A_4^4 = 1100 \longrightarrow g(A_4^4) = 6.0 \longrightarrow F(A_4^4) = 0.1000 \longrightarrow P(A_4^4) = 0.0881$$

After reaching this point, the algorithm could be stopped since the optimum value obtained in future generations will, most probably, be unchanged due to the high reproduction probability of the fittest chromosome yielding $x = 4$. Then there should not be any reasons to continue the search. However, as opposed to this steady state, there might be some points in the algorithm where the search is stuck at some not very desirable value. Then, mutations can be initiated into the system to avoid these problems.

It should be noted, that as stated by one of the pioneers of evolutionary algorithms, Fogel (1999), GA's possess further capabilities to solve much more complex problems than the one above. He asserts that the ease of their implementation and the broad range of areas where they are applicable are very valuable characteristics for those who work with very unusual functional forms. Usual methods of optimization used in economics generally require functions of continuous, differentiable and, even, convex nature in order to serve their purpose properly. However, there are classes of problems which none of these conditions might hold. The objective function might be discrete and moreover it might have local extrema, rendering some gradient based solutions infeasible. Non-linear constraints and non-stationary conditions in real world problems also increase computational burden making it difficult to use some of the conventional techniques. Genetic algorithms are thus useful in solving complex optimization problems with such characteristics without necessitating simplifying assumptions about functional forms.

A GA model is particularly suitable for the problem in this thesis since the objective function is discrete and the feasible region is not easy to visualize. Still, it is certain that there is some kind of monotonic behavior expected of the objective function in response to increases or decreases in pension parameter values. So, even though numerous local maxima and minima may be encountered, GA developed here must be capable of handling them.

With its ease of application to economics problems, and its robustness with discrete functions, the GA's have become a useful tool for economists. The pioneering work in the field is done by Arifovic (1994) where she simulates learning in a rational expectations equilibrium model. In the model, competitive firms use a genetic algorithm to update their decision on which and what amount of good to use the next period in a one good production economy.

In another paper, Arifovic (1995) presents a two-period overlapping generations (OLG) model with money, low inflation and seignorage to show that the individuals' policies dictated by learning through GA are the same as the ones at unique monetary steady state equilibrium. Bullard and Duffy (1998) extend this model to a multi-period OLG model to find out a high inflation equilibrium, a low inflation equilibrium as well as undecided cases. Another application of GA's in economics is the game theoretical solution search where a game is modeled as a GA and the possible equilibria are searched for. An example to this application is Özyıldırım (1997). The paper approximates solutions to non-linear non-quadratic open loop Nash-Cournot equilibrium, where agents involved in a multi-period game creating their own policies about what to do at the beginning of each period at time 0 without cooperating with each other. Once the policy is made in time 0, the agents stick to their decisions and the game is played accordingly. The result is the case where none of the agents can deviate from the equilibrium policy without getting worse off. For a two-person game, the solution starts out by creating a GA structure for each player. Each GA structure has the player's utility function, and population. At the beginning, each player sends out a random strategy to the other player, which the other player accepts as the best response to his action. With the best response at hand, players then move on to create new strategies from the acquired best response functions and their previous moves. The process works as the initial population is ranked with the help of the evaluation function and cross-overs and mutations are performed in the selected group. At the end of the selection process, a new strategy is formed and sent to the other player, initiating the next round. The algorithm terminates once the termination criteria are satisfied.

In another study, Alemdar and Özyıldırım (1998) use GA's to search for optimal policies in a North/South trading game.

Another interesting use to GA's in economics/finance can be found in Chen and Yeh (2002) who use GA's to model agents in a stock market setting through an “agent-based economics” approach which uses GA’s heavily to simulate a real life situation.

CHAPTER 3

THE MODEL

3.1 The Objective Function

The current Turkish pension system is run on a pay-as-you-go basis which finances the payments made to current pensioners through contributions collected from current workers. The Social Security Institution therefore needs to bring present value of its future receipts as close to the present value of its commitments as possible to run the system without significant deficit or surplus.

In the model, the time horizon is taken to be the period between years 2005 and 2060, when Turkey's population is expected to have stabilized and reached its steady state at less than 100 million people. Thus, the payments-receipts balance achieved by 2060 will represent a reasonable approximation to the SSI's deficit level for 2061 and onwards.

One way of restoring the long-term actuarial balance of the SSI is to minimize the following expression, mostly adapted from Sayan and Turhan-Sayan (2001).

$$\min_{RR,CR,A}\left|\sum_{t=2005}^{t=2060}\left(\frac{1}{1+\delta}\right)^{t-2005}\left[RR_t\sum_{a=A_t}^{le}\overline{rw_{a,t}}R_{a,t}(RR,CR,A)-CR_t\sum_{a=a_0}^{mwa}rw_{a,t}W_{a,t}(RR,CR,A)\right]\right|$$

s.t. (1)

Retirement Age Constraints:

$\forall a : 41 \le a < A_t,\ \forall t : 2005 \le t \le 2060 :$

$$W_{a,t}(RR^o, CR^o, A) = W_{a,t}(RR^o, CR^o, A^o) + R_{a,t}(RR^o, CR^o, A^o) \tag{2}$$

$$R_{a,t}(RR^o, CR^o, A) = 0 \tag{3}$$

$\forall a : 75 \ge a \ge A_t,\ \forall t : 2005 \le t \le 2060 :$

$$W_{a,t}(RR^o, CR^o, A) = W_{a,t}(RR^o, CR^o, A^o) \tag{4}$$

$$R_{a,t}(RR^o, CR^o A) = R_{a,t}(RR^o, CR^o, A^o) \tag{5}$$

Income Elasticity of Labor and Pension Constraints:

Constraints for policy change in the initial year of the model:

$\forall a : A_t \le a < mwa,\ \forall t : 2005 \le t \le 2060$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2005}, \{RR_{\tau'}^o\}_{\tau'=2006}^{\tau'=2060}, \{CR_\tau\}_{\tau=2005}^{\tau=2005}, \{CR_\tau^o\}_{\tau=2006}^{\tau=2060}, A) =$$

$$W_{a,t}(RR^o, CR^o, A) + DW_{a,t}(RR^o, CR^o, A) \tag{6}$$

$$\mathrm{R}_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2005}, \{RR_{\tau'}^o\}_{\tau'=2006}^{\tau'=2060}, \{CR_\tau\}_{\tau=2005}^{\tau=2005}, \{CR_\tau^o\}_{\tau=2006}^{\tau=2060}, A) =$$

$$\mathrm{R}_{a,t}(RR^o, CR^o, A) - DW_{a,t}(RR^o, CR^o, A) \tag{7}$$

Constraints for policy change for years 2006 through 2059:

$\forall a : A_t \le a < mwa,\ \forall t < \tau,\ t : 2005 \le t \le 2060,\ \forall \tau : 2006 \le \tau \le 2060$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau}, \{RR_{\tau'}^o\}_{\tau'=\tau+1}^{\tau'=2060}, \{CR_\tau\}_{\tau'=2005}^{\tau'=\tau}, \{CR_\tau^o\}_{\tau'=\tau+1}^{\tau'=2060}, A) =$$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1}, \{RR_{\tau'}^o\}_{\tau'=\tau}^{\tau'=2060}, \{CR_\tau\}_{\tau'=2005}^{\tau'=\tau-1}, \{CR_\tau^o\}_{\tau'=\tau}^{\tau'=2060}, A) \tag{8}$$

$$\mathrm{R}_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau}, \{RR_{\tau'}^o\}_{\tau'=\tau+1}^{\tau'=2060}, \{CR_\tau\}_{\tau'=2005}^{\tau'=\tau}, \{CR_\tau^o\}_{\tau'=\tau+1}^{\tau'=2060}, A) =$$

$$\mathrm{R}_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1},\{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1},\{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060},A) \tag{9}$$

$$\forall a : A_t \leq a < mwa,\ \forall t \geq \tau,\ t : 2005 \leq t \leq 2060,\ \forall \tau : 2006 \leq \tau \leq 2060$$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau},\{RR_{\tau'}^{o}\}_{\tau'=\tau+1}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau},\{CR_{\tau}^{o}\}_{\tau'=\tau+1}^{\tau'=2060},A) =$$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1},\{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1},\{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060},A) +$$

$$DW_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1},\{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1},\{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060},A) \tag{10}$$

$$\mathrm{R}_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau},\{RR_{\tau'}^{o}\}_{\tau'=\tau+1}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau},\{CR_{\tau}^{o}\}_{\tau'=\tau+1}^{\tau'=2060},A) =$$

$$\mathrm{R}_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1},\{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1},\{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060},A) -$$

$$DW_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1},\{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1},\{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060},A) \tag{11}$$

Constraints for the final year of the model:

$$\forall a : A_t \leq a < mwa,\ \forall t < \tau,\ t : 2005 \leq t \leq 2059$$

$$W_{a,t}(RR,CR,A) =$$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2059},\{RR_{\tau'}^{o}\}_{\tau'=2060}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=2059},\{CR_{\tau}^{o}\}_{\tau'=2060}^{\tau'=2060},A) \tag{12}$$

$$R_{a,t}(RR,CR,A) =$$

$$R_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2059},\{RR_{\tau'}^{o}\}_{\tau'=2060}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=2059},\{CR_{\tau}^{o}\}_{\tau'=2060}^{\tau'=2060},A) \tag{13}$$

$$\forall a : A_t \leq a < mwa,\ t = 2060$$

$$W_{a,t}(RR,CR,A) =$$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2059},\{RR_{\tau'}^{o}\}_{\tau'=2060}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=2059},\{CR_{\tau}^{o}\}_{\tau'=2060}^{\tau'=2060},A) -$$

$$DW_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2059},\{RR_{\tau'}^{o}\}_{\tau'=2060}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=2059},\{CR_{\tau}^{o}\}_{\tau'=2060}^{\tau'=2060},A) \tag{14}$$

$$\mathrm{R}_{a,t}(RR,CR,A) =$$

$$\mathrm{R}_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2059},\{RR_{\tau'}^{o}\}_{\tau'=2060}^{\tau'=2060},\{CR_{\tau}\}_{\tau'=2005}^{\tau'=2059},\{CR_{\tau}^{o}\}_{\tau'=2060}^{\tau'=2060},A) -$$

$$DW_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2059}, \{RR_{\tau'}^{o}\}_{\tau'=2060}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=2059}, \{CR_{\tau}^{o}\}_{\tau'=2060}^{\tau'=2060}, A) \qquad (15)$$

and

$$DW_{a,t}(\cdot) = \begin{cases} -\frac{1}{2}\left[\lambda \frac{W_{a,t}(\cdot)\, dCR_t}{1-CR_t} + \lambda \frac{R_{a,t}(\cdot) dRR_t}{RR_t}\right] & \text{, if } -\frac{1}{2}\left[\lambda \frac{W_{a,t}(\cdot) dCR_t}{1-CR_t} + \lambda \frac{R_{a,t}(\cdot) dRR_t}{RR_t}\right] < 0 \\ 0 & \text{, else} \end{cases} \qquad (16)$$

where

CR_t : Average contribution rate per worker in year t ($0 < CR_t < 1$),

RR_t : Average replacement rate of a pensioner in year t to replace average work income ($0 < RR_t < 1$)

A_t : Minimum retirement age in year t ($A_t < mwa$),

RR, CR, A : $\{RR_t\}_{t=2005}^{t=2060}, \{CR_t\}_{t=2005}^{t=2060}, \{A_t\}_{t=2005}^{t=2060}$

$W/R_{a,t}(RR, CR, A)$: Projected number of workers/pensioners of age a in year t as a function of pension parameters in the policy matrix, $\{RR, CR, A\}$,

$W/R_{a,t}(RR^o, CR^o, A^o)$: Original projection of number of workers/pensioners of age a in year t as a function of pre-1999 pension parameters, $\{RR^o, CR^o, A^o\}$,

$W/R_{a,t}(RR^o, CR^o, A)$: Projection of number of workers/pensioners of age a in year t updated according to the new value of A only,

$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau}, \{RR_{\tau'}^{o}\}_{\tau'=\tau+1}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau}, \{CR_{\tau}^{o}\}_{\tau'=\tau+1}^{\tau'=2060}, A)$: A worker population matrix showing the steps in transforming the final form of retirement age constraint, $W_{a,t}(RR^o, CR^o, A)$, to the final elasticity constrained worker matrix, $W_{a,t}(RR, CR, A)$, where τ represents the year up to which the transformation has successfully been completed.

$\overline{rw}_{a,t}$: Average real work income of pensioners of age a in year t,

$rw_{a,t}$: Average real work income of workers of age a in year t,

a_0 :Minimum working age,

le :Life expectancy.

λ :Income elasticity constant,

ρ : Pension elasticity constant,

mwa : Maximum working age,

δ : Discount factor,

The objective function to be minimized describes the difference between the total present discounted values of contribution receipts and pension payments to face the SSI over the model horizon, provided that pension parameters are $\{RR^o, CR^o, A^o\}$. Total wages earned by the workers covered by the SSI in a given year t is calculated by multiplying the number of workers in a given cohort, $W_{a,t}(RR, CR, A)$, by the average wage of workers belonging to that cohort, $rw_{a,t}$, and then summing the result over all age groups in year t, $\sum_{a=mwa}^{le} rw_{a,t} W_{a,t}(RR, CR, A)$. Total contribution revenue of the SSI for a given year t is calculated by multiplying this sum with the contribution rate for that year, CR_t. The resulting value is converted to the present value terms by a standard present value operator to yield, $\left(\frac{1}{1+\delta}\right)^{t-2005} CR_t \sum_{a=a_0}^{mwa} rw_{a,t} W_{a,t}(RR, CR, A)$. Then, the present values are summed over for all possible years, to obtain total present value of the future receipts of the SSI, $\sum_{t=2005}^{t=2060} \left(\frac{1}{1+\delta}\right)^{t-2005} CR_t \sum_{a=a_0}^{mwa} rw_{a,t} W_{a,t}(RR, CR, A)$

Total present value of the future pension expenditure of the SSI is obtained in a similar way resulting in $\sum_{t=2005}^{t=2060}\left(\frac{1}{1+\delta}\right)^{t-2005} RR_t \sum_{a=A_t}^{le} \overline{rw_{a,t}} R_{a,t}(RR, CR, A)$.

Therefore, subtracting the expenditures from revenues, we get the present value of the total deficit/surplus. Normally, a publicly managed pension fund would not aim at high surpluses but would ideally avoid high deficits. Naturally, the deficit/surplus value should be as close to zero as possible. This is achieved in the model by using the absolute value operator whose global minimum is naturally zero.

3.2. The Constraints

The optimization discussed up to this point requires an extensive knowledge of the labor force covered by SSI, which is represented by the following data:

i. The projections of the number of workers, $W_{a,t}(RR^o, CR^o, A^o)$, for each age *a* (where *a* takes integer values 15 through 75 plus) and for each year *t* (where *t* takes integer values 2005 through 2060),

ii. The projections of the number of retirees, $R_{a,t}(RR^o, CR^o, A^o)$, for each age *a* (where a takes integer values 41 through 80 plus) and for each year *t* (where *t* takes integer values 2005 through 2060),

iii. The projections of the average annual real work incomes of workers, $rw_{a,t}$, for each age *a* (where *a* takes integer values 15 through 75 plus) and for each year *t* (where *t* takes integer values 2005 through 2060),

iv. The projections of the average annual real work incomes of retirees, $\overline{rw}_{a,t}$, for each cohort a (where a takes integer values 41 through 80 plus) for each year t (where t takes integer values 2005 through 2060),

where the plus sign following an integer represents the set of all integers after that integer.

Initially, four projection matrices, $W_{a,t}(RR^o, CR^o, A^o)$, $R_{a,t}(RR^o, CR^o, A^o)$, $rw_{a,t}$, and $\overline{rw}_{a,t,o}$, for items (a) through (d) above are available through ILO (1996) and Sayan and Turhan-Sayan (2001). These matrices contain projections that are based on pre-1999 values of pension parameters. Therefore, they include non-zero values for the number of retirees for all years in the model for ages as low as 38.

The results of our optimization exercises therefore rely heavily on these projections but the projected values of gross real wages will assume not to be affected by the changes brought forth by alternative reform parameters.

Such dependence creates the need to update the original projections on the numbers of workers and retirees accordingly with the changes in pension parameters. This is achieved by using the retirement age and elasticity constraints.

3.2.a Retirement Age Constraint

The first step in updating projections based on the pre-1999 values of pension parameters is the creation of worker/retiree projections in accordance with the proposed increases in retirement ages, A. The retirement age vector A is designed such that its first element corresponds to the proposed minimum retirement age in year 2005, and its second element corresponds to the retirement age in year 2006. Proceeding like this, the last element of the vector would be matched with the last year in the model horizon, 2060.

The elements of this vector take monotonically increasing values since it is assumed that age is increased from one year to another, it will not be reduced ever again in the future. This is a reasonable assumption since the life expectancy of the population is also expected to increase in the future.

The algorithm starts out from year 2005 which is the first element in *A*. The retirement age is checked and found to be, say, a_1. This means that the retirees who are below age a_1 in 2005 will no longer be able to retire as dictated in the original retiree matrix and will have to work until the newly posted retirement age. These people are consequently transferred to the labor force, *W*, increasing the number of workers at age a_1 in year 2005 by the number of retirees at age a_1 in the same year. Naturally, this brings down the number of retirees at that age in that year strictly to zero. The new labor force values after a proposed 2005 retirement age policy change are presented below.

$\forall a : 41 \le a < A_t,\ \forall t : 2005 \le t \le 2060 :$

$$W_{a,t}(RR^o, CR^o, A) = W_{a,t}(RR^o, CR^o, A^o) + R_{a,t}(RR^o, CR^o, A^o) \qquad (17)$$

$$R_{a,t}(RR^o, CR^o, A) = 0 \qquad (18)$$

$\forall a : 75 \ge a \ge A_t,\ \forall t : 2005 \le t \le 2060 :$

$$W_{a,t}(RR^o, CR^o, A) = W_{a,t}(RR^o, CR^o, A^o) \qquad (19)$$

$$R_{a,t}(RR^o, CR^o, A) = \mathrm{R}_{a,t}(RR^o, CR^o, A^o) \qquad (20)$$

Repeating the same steps for retirement ages for years 2006 to 2060 yields the new worker and retiree projections, $W_{a,t}(RR^o, CR^o, A)$ and $R_{a,t}(RR^o, CR^o, A)$. The following pictorial aid depicting the process might be helpful.

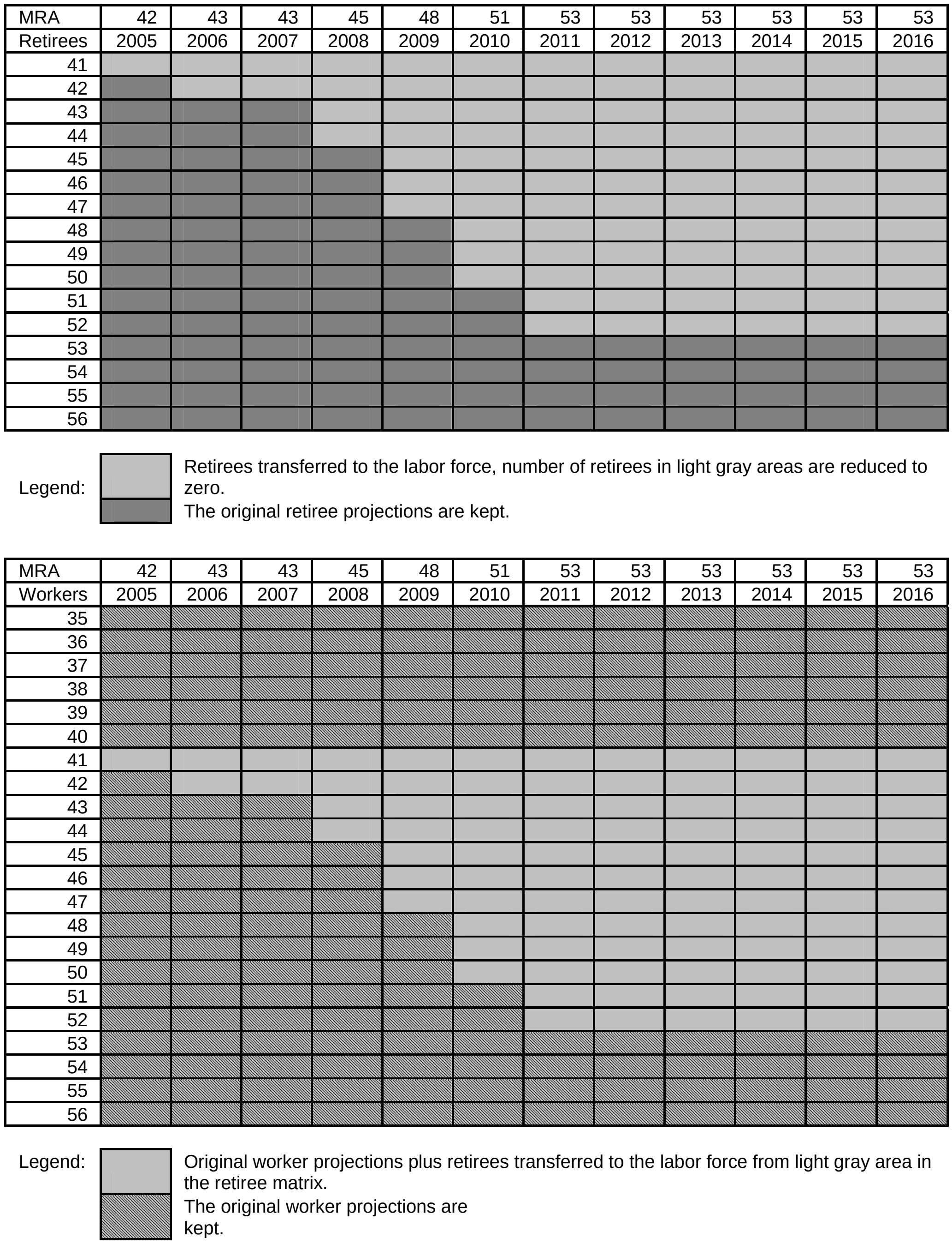

Figure 3: Retirement Age Constraint Pictorial Aid

Furthermore, a flowchart sketching the algorithm is shown below.

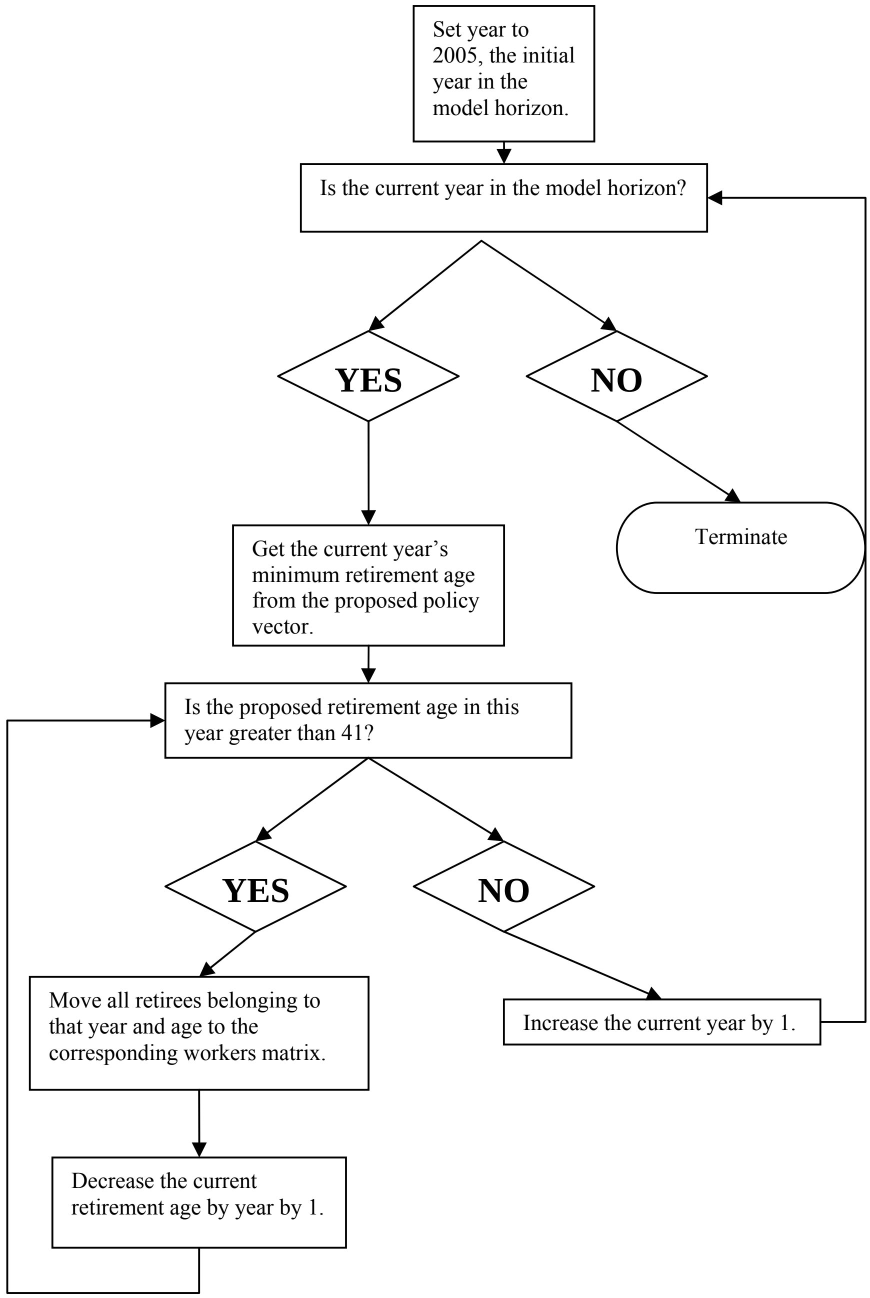

Figure 4: Retirement Age Constraint Implementation Algorithm

The matrices updated to take the retirement age constraint into account are used in calculating the additional changes to result from changes in CR and RR.

3.2.b Work and Pension Income Elasticities of Labor Supply Constraints

In the model, the disposable income of a worker is calculated by multiplying the real annual wage income of the worker by (1-*CR*), the percentage of income received by the worker net of contributions paid to the SSI. That is $I = (1 - CR_t) rw_{a,t}$.

Similarly, the pension income is calculated by multiplying the average real annual wage income of the pensioner over the working period by RR, the replacement rate: $P = RR_t rw_{a,t}$.

In the light of these definitions, we can consider the response of individuals who are between the statutory entitlement age and maximum working age to changes in CR and/or RR. Since these individuals have the option of continuing to work or taking their retirement depending upon the incentives, changes in CR and RR would affect the worker-retiree composition of this age group.

Now, let λ be the income elasticity of labor supply. Then,

$$\lambda = \frac{\dfrac{dW_{a,t}(\cdot)}{W_{a,t}(\cdot)}}{\dfrac{dWI_{a,t}}{WI_{a,t}}} \tag{21}$$

where work income is $WI_{a,t} = (1 - CR_t) rw_{a,t}$ and $A_t \leq a \leq mwa$. Naturally, $\lambda > 0$, and its definition can be rearranged to yield $\dfrac{dW_{a,t}(\cdot)}{dWI_{a,t}} = \lambda \dfrac{W_{a,t}(\cdot)}{WI_{a,t}}$.

Note that

$$\frac{dW_{a,t}(\cdot)}{dWI_{a,t}} \equiv \frac{dW_{a,t}(\cdot)}{dCR_t} \frac{dCR_t}{dWI_{a,t}}$$

$$\equiv \frac{dW_{a,t}(\cdot) / dCR_t}{dWI_{a,t} / dCR_t}$$

$$\equiv -\frac{dW_{a,t}(\cdot) / dCR_t}{rw_{a,t}} \tag{22}$$

Combining this with (21), we can write

$$\frac{dW_{a,t}(\cdot)}{dCR_t} = -\lambda \frac{W_{a,t}(\cdot) rw_{a,t}}{WI_{a,t}} = -\lambda \frac{W_{a,t}(\cdot)}{(1-CR_t)} \tag{23}$$

$$= \frac{dW_{a,t}(\cdot) / W_{a,t}(\cdot)}{dCR_t / (1-CR_t)} = -\lambda \tag{24}$$

Equation 24 enabled us to convert income elasticity of labor supply into a contribution rate elasticity. This equation could alternatively be written as

$$\frac{W_{a,t+1}(\cdot) - W_{a,t}(\cdot)}{W_{a,t}(\cdot)} \quad = -\lambda \frac{CR_{t+1} - CR_t}{(1-CR_t)} \tag{25}$$

with discrete changes in percentage terms on both sides, allowing us to interpret λ as net wage elasticity of labor supply. Corresponding to λ is an RR elasticity that allows us to capture the response of individuals aged a at time t to changes in RR. Let this elasticity be denoted by ρ and defined as

$$\rho = \frac{\dfrac{dR_{a,t}(\cdot)}{R_{a,t}(\cdot)}}{\dfrac{dPI_{a,t}}{PI_{a,t}}} \tag{26}$$

where $PI_t = RR_t \overline{rw_{a,t}}$ and $\rho > 0$. Then,

$$\frac{dR_{a,t}(\cdot)}{R_{a,t}(\cdot)} = \rho \frac{dPI_{a,t}}{PI_{a,t}} = \rho \frac{dRR_t \overline{rw}_{a,t}}{RR_t \overline{rw}_{a,t}} \tag{27}$$

Thus, ρ shows the percentage change in the number of retiree resulting from a one percent increase in the replacement rate. Equation 27 can be equivalently written as

$$\frac{R_{a,t+1}(\cdot) - R_{a,t}(\cdot)}{R_{a,t}(\cdot)} = -\rho \frac{RR_{t+1} - RR_t}{(1 - RR_t)} \tag{28}$$

Now, remembering that the population, $p_{a,t}$, for any particular year and age is constant regardless of the policy changes, $R_{a,t}(\cdot) + W_{a,t}(\cdot) = p_{a,t}$

$$\frac{dW_{a,t}(\cdot)}{dRR_t} = \frac{dp_{a,t}}{dRR_t} - \frac{dR_{a,t}(\cdot)}{dRR_t} \tag{29}$$

Since $p_{a,t}$ is constant, we can write the following by keeping Equation 27 in mind.

$$\frac{dW_{a,t}(\cdot)}{dRR_t} = -\frac{dR_{a,t}(\cdot)}{dRR_t}$$

$$= -\rho \frac{R_{a,t}(\cdot)}{RR_t} \tag{30}$$

Equation 24 and Equation 30 together imply

$$2dW_{a,t}(\cdot) = -\left[\lambda \frac{W_{a,t}(\cdot)dCR_t}{1 - CR_t} + \rho \frac{R_{a,t}(\cdot)dRR_t}{RR_t}\right]$$

or

$$dW_{a,t}(\cdot) = -\frac{1}{2}\left[\lambda \frac{W_{a,t}(\cdot)dCR_t}{1 - CR_t} + \rho \frac{R_{a,t}(\cdot)dRR_t}{RR_t}\right] \tag{31}$$

in which $dW_{a,t}$ may be positive or negative depending on the magnitudes of changes in CR and RR.

However, it should be noted that the changes that demand an increase in the number of workers (such as a reduction in CR) while keeping the registered population constant cannot be met. This is because the retirees who by definition have quit contributing to the system cannot be returned to the workforce. Then, increasing the number of workers without reducing the number of retirees would require increasing the projected number of people within the age group under consideration. Hence, we assume that transfers from projected workforce matrices to the projected retiree matrices are unidirectional in the sense that only workers can be transferred to the retiree matrices. Then, the change in the number of workers for any change in CR and RR would be

$$DW_{a,t}(\cdot) = \begin{cases} dW_{a,t}(\cdot) & \text{, if } dW_{a,t}(\cdot) < 0 \\ 0 & \text{, else} \end{cases} \tag{32}$$

Projected numbers of workers and retirees are updated in response to changes in CR and RR as follows:

Initially, the worker and retiree projections that are based on the current values of contribution and replacement rates are manipulated to address minimum retirement age changes. From now on, the resulting matrices will be referred to as $W_{a,t}(RR^o, CR^o, A)$ and $R_{a,t}(RR^o, CR^o, A)$, where the replacement of A^o by A indicates that minimum retirement age modification is already completed on the matrix, whereas keeping RR^o and CR^o indicates that no modifications on account of changes in CR and RR have been introduced to matrices yet.

Once a change is introduced to the contribution and replacement rates in the first year of the model, 2005, the worker retiree matrices must be adjusted so as to obtain new matrices based on the new contribution and replacement rates for the year 2005. That is, $W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2005}, \{RR_{\tau'}^o\}_{\tau'=2006}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=2005}, \{CR_{\tau}^o\}_{\tau'=2006}^{\tau'=2060}, A)$ and

$R_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2005}, \{RR_{\tau'}^{o}\}_{\tau'=2006}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=2005}, \{CR_{\tau}^{o}\}_{\tau'=2006}^{\tau'=2060}, A)$. This is achieved by applying the elasticity formula above to transfer some of the workers projected to stay in the workforce under old values of CR and RR to the projected retiree population for all of the remaining years in the model. After making the necessary worker transfers, the transferred workers are subtracted from the retiree matrix to keep the population constant.

$\forall a : A_t \leq a < mwa,\ \forall t : 2005 \leq t \leq 2060$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2005}, \{RR_{\tau'}^{o}\}_{\tau'=2006}^{\tau'=2060}, \{CR_{\tau}\}_{\tau=2005}^{\tau=2005}, \{CR_{\tau}^{o}\}_{\tau=2006}^{\tau=2060}, A) =$$

$$\mathrm{W}_{a,t}(RR^{o}, CR^{o}, A) + DW_{a,t}(RR^{o}, CR^{o}, A) \tag{33}$$

$$\mathrm{R}_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2005}, \{RR_{\tau'}^{o}\}_{\tau'=2006}^{\tau'=2060}, \{CR_{\tau}\}_{\tau=2005}^{\tau=2005}, \{CR_{\tau}^{o}\}_{\tau=2006}^{\tau=2060}, A) =$$

$$\mathrm{R}_{a,t}(RR^{o}, CR^{o}, A) + DW_{a,t}(RR^{o}, CR^{o}, A) \tag{34}$$

As a consequence, the change injected into the worker retiree projections becomes a once-and-for-all change affecting all present and future numbers of retirees.

Once the data is updated like this, the next year's projections, $R_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2006}, \{RR_{\tau'}^{o}\}_{\tau'=2007}^{\tau'=2060}, \{CR_{\tau}\}_{\tau=2005}^{\tau=2006}, \{CR_{\tau}^{o}\}_{\tau=2007}^{\tau=2060}, A)$ and $W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=2006}, \{RR_{\tau'}^{o}\}_{\tau'=2007}^{\tau'=2060}, \{CR_{\tau}\}_{\tau=2005}^{\tau=2006}, \{CR_{\tau}^{o}\}_{\tau=2007}^{\tau=2060}, A)$, will be created similarly whenever there is a change in the policy. In general, for an arbitrary year τ in the horizon, in the case of a policy change, relevant projections are updated as follows. First of all, the process starts out by noting that current and future policies cannot affect the number of workers and retirees projected for the previous years. Hence, these values become natural elements of current matrices. Then, the change in current policy variables is applied to every age group in every year for the remaining years in the horizon to update the matrices in accordance with the policy changes. The algorithm thus moves forward to cover all years in the model horizon. A general approach is presented below.

$$\forall a : A_t \le a < mwa,\ \forall t < \tau,\ t : 2005 \le t \le 2060,\ \forall \tau : 2006 \le \tau \le 2060$$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau}, \{RR_{\tau'}^{o}\}_{\tau'=\tau+1}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau}, \{CR_{\tau}^{o}\}_{\tau'=\tau+1}^{\tau'=2060}, A) =$$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1}, \{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1}, \{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060}, A) \tag{35}$$

$$\forall a : A_t \le a < mwa,\ \forall t \ge \tau,\ t : 2005 \le t \le 2060,\ \forall \tau : 2006 \le \tau \le 2060$$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau}, \{RR_{\tau'}^{o}\}_{\tau'=\tau+1}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau}, \{CR_{\tau}^{o}\}_{\tau'=\tau+1}^{\tau'=2060}, A) =$$

$$W_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1}, \{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1}, \{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060}, A) +$$

$$DW_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1}, \{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1}, \{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060}, A) \tag{36}$$

$$\forall a : A_t \le a < mwa,\ \forall t < \tau,\ t : 2005 \le t \le 2060,\ \forall \tau : 2006 \le \tau \le 2060$$

$$\mathrm{R}_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau}, \{RR_{\tau'}^{o}\}_{\tau'=\tau+1}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau}, \{CR_{\tau}^{o}\}_{\tau'=\tau+1}^{\tau'=2060}, A) =$$

$$\mathrm{R}_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1}, \{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1}, \{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060}, A) \tag{37}$$

$$\forall a : A_t \le a < mwa,\ \forall t \ge \tau,\ t : 2005 \le t \le 2060,\ \forall \tau : 2006 \le \tau \le 2060$$

$$R_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau}, \{RR_{\tau'}^{o}\}_{\tau'=\tau+1}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau}, \{CR_{\tau}^{o}\}_{\tau'=\tau+1}^{\tau'=2060}, A) =$$

$$\mathrm{R}_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1}, \{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1}, \{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060}, A) -$$

$$DW_{a,t}(\{RR_{\tau'}\}_{\tau'=2005}^{\tau'=\tau-1}, \{RR_{\tau'}^{o}\}_{\tau'=\tau}^{\tau'=2060}, \{CR_{\tau}\}_{\tau'=2005}^{\tau'=\tau-1}, \{CR_{\tau}^{o}\}_{\tau'=\tau}^{\tau'=2060}, A) \tag{38}$$

The data for the final year of the model horizon can also be calculated from the previous year's data in a similar way.

Following is the algorithm for implementing the elasticity constraints.

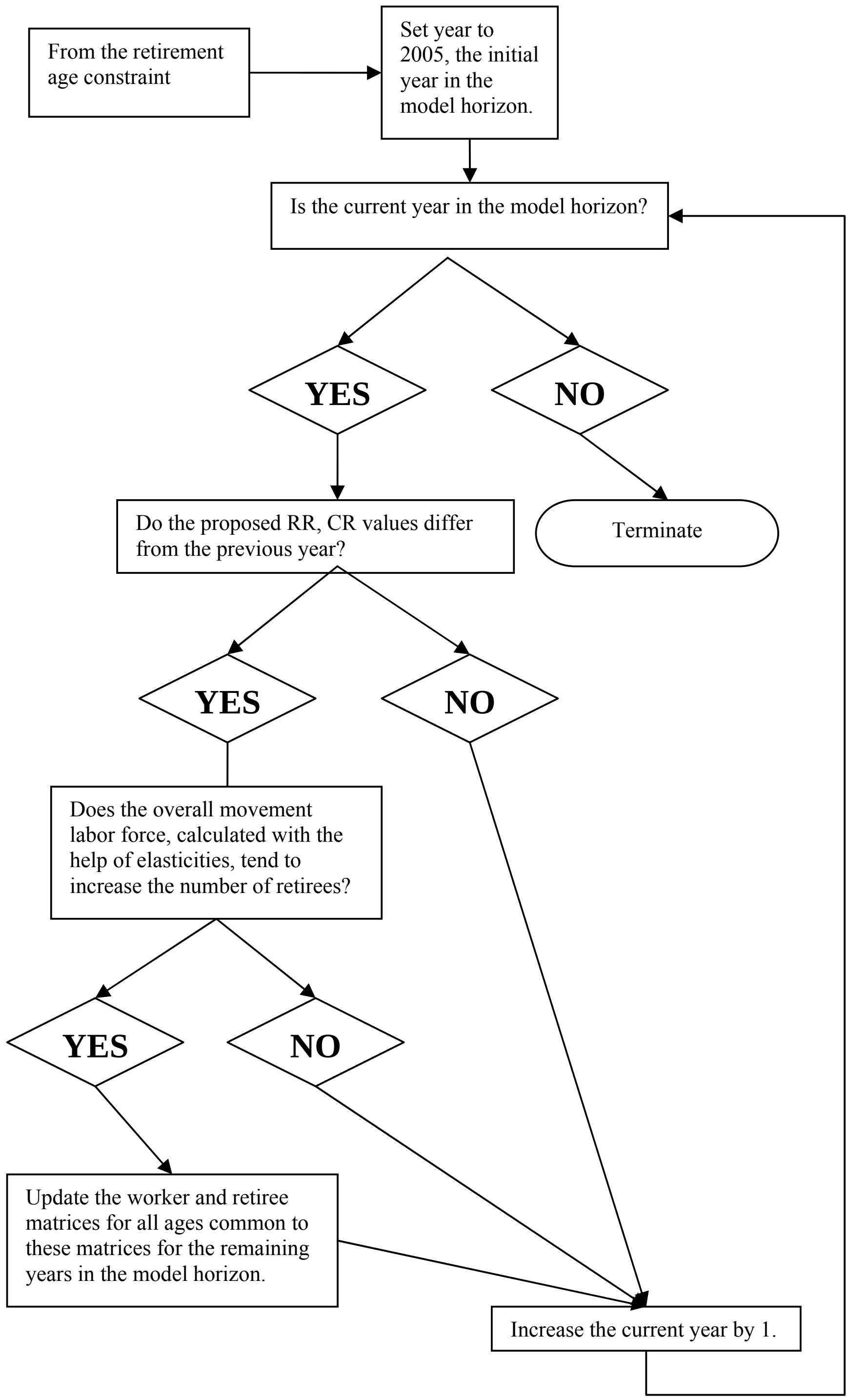

Figure 5: Elasticity Constraints Implementation Algorithm

The program that translates the effects of changes in the minimum retirement age and contribution/replacement rates into the model becomes a part of the fitness calculating code to order the chromosomes according to their usefulness.

CHAPTER 4

RESULTS

The aim of this section is to compare results obtained with the GA to the baseline scenarios of the 1999 Act and its 2002 modification required by the Turkish Constitutional Court as well as the current social security proposal reform bill expected to be legislated in 2006.

In the rest of the chapter, the constants used in the optimization program take the following values. The labor elasticity of income and pension are taken to be 0.2, while the discount factor is 0.05. Also, for any further inquiries about the code used in optimization, the MATLAB code written for the solution of the problem is presented in the Appendix.

First of all, the parametric adjustments in 1999, 2002, and 2006 (expected) are given below.

Table 1: Parametric Adjustments in 1999, 2002, and 2006 (Expected)

	1999			2002			2006 Proposal		
Year	Min. Ret. Age	RR	CR	Min Ret. Age	RR	CR	Min. Ret. Age	RR	CR
2005	48	0.65	0.20	41	0.65	0.20	41	0.65	0.20
2006	49	0.65	0.20	42	0.65	0.20	42	0.625	0.20
2007	50	0.65	0.20	43	0.65	0.20	43	0.625	0.20
2008	51	0.65	0.20	44	0.65	0.20	44	0.625	0.20
2009	58	0.65	0.20	45	0.65	0.20	45	0.625	0.20
2010	58	0.65	0.20	46	0.65	0.20	46	0.625	0.20
2011	58	0.65	0.20	47	0.65	0.20	47	0.625	0.20
2012	58	0.65	0.20	48	0.65	0.20	48	0.625	0.20
2013	58	0.65	0.20	49	0.65	0.20	49	0.625	0.20
2014	58	0.65	0.20	50	0.65	0.20	50	0.625	0.20
2015	58	0.65	0.20	51	0.65	0.20	51	0.625	0.20

Table 1: Cont'd

2016	58	0.65	0.20	52	0.65	0.20	52	0.625	0.20
2017	58	0.65	0.20	53	0.65	0.20	53	0.50	0.20
2020	58	0.65	0.20	56	0.65	0.20	56	0.50	0.20
2021-2060	58	0.65	0.20	58	0.65	0.20	58	0.50	0.20

Calculations based on these parameter specifications yield the following accumulated deficits for the model duration.

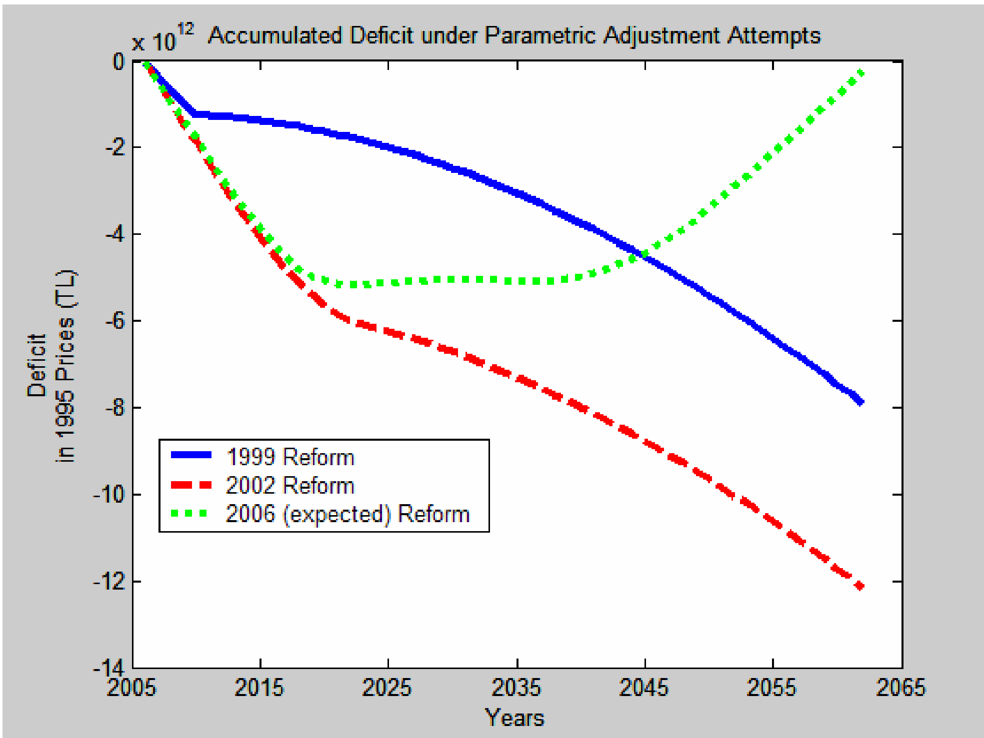

Figure 6: Accumulated Deficit under Parametric Adjustment attempts

The scenarios considered here are as follows.

4.1 Scenario 1

In this exercise, the three components of the policy reform are subjected to change in five to ten year intervals.

The minimum retirement age structure is kept in accordance with the 2002 adjustment until year 2020, and then gradual increases in the minimum retirement age are allowed for every 5-year period. The changes are generally binary choices leading the program to make a choice between two consecutive retirement ages until year 2050, after which interval is considered for possible increases in the minimum retirement age. Finally, the 2055 policy change allows the computer to make a choice between age 65 and 66 to decide on the final retirement age in the model. The difference from the current proposal lies in its flexibility to allow 65, a lower value than the current proposal's 66 to be the minimum retirement age in 2060. Furthermore, the increases in retirement ages span 5 year terms to bring the policy change years approximately in the middle of two consecutive elections to avoid any populist moves.

The RR and CR values are allowed to float in the region 0.65 to 0.725 for RR and 0.20 to 0.275 for CR. The policy change occurs in every 10 years to accommodate changes in the age structure to reach the optimum long-run deficit values. The policy change years are also selected to be in the middle of two consecutive elections to avoid any populist moves.

The allowed ranges of pension parameters and the GA output are presented below in Table 2.

Table 2: Policy Proposal and the GA Output for Scenario 1

	Allowed Ranges of Pension Parameters									Results		
Year	min MRA	max MRA	Bit Depth	min. RR	max. RR	Bit Depth	CR	max. CR	Bit Depth	MRA	RR	CR
2005	41	41	0	0.65	0.725	4	0.2	0.275	4	41	0.65	0.245
2006	42	42	0	0.65	0.725	»	0.2	0.275	»	42	0.65	0.245
2007	43	43	0	0.65	0.725	»	0.2	0.275	»	43	0.65	0.245
2008	44	44	0	0.65	0.725	»	0.2	0.275	»	44	0.65	0.245
2009	45	45	0	0.65	0.725	»	0.2	0.275	»	45	0.65	0.245
2010	46	46	0	0.65	0.725	»	0.2	0.275	»	46	0.65	0.245
2011	47	47	0	0.65	0.725	»	0.2	0.275	»	47	0.65	0.245
2012	48	48	0	0.65	0.725	»	0.2	0.275	»	48	0.65	0.245
2013	49	49	0	0.65	0.725	»	0.2	0.275	»	49	0.65	0.245
2014	50	50	0	0.65	0.725	»	0.2	0.275	»	50	0.65	0.245
2015	51	51	0	0.65	0.725	4	0.2	0.275	4	51	0.655	0.27

Table 2: Con't

2016	52	52	0	0.65	0.725	»	0.2	0.275	»	52	0.655	0.27
2017	53	53	0	0.65	0.725	»	0.2	0.275	»	53	0.655	0.27
2018	54	54	0	0.65	0.725	»	0.2	0.275	»	54	0.655	0.27
2019	55	55	0	0.65	0.725	»	0.2	0.275	»	55	0.655	0.27
2020	56	56	0	0.65	0.725	»	0.2	0.275	»	56	0.655	0.27
2021	56	56	0	0.65	0.725	»	0.2	0.275	»	56	0.655	0.27
2022	56	56	0	0.65	0.725	»	0.2	0.275	»	56	0.655	0.27
2023	56	56	0	0.65	0.725	»	0.2	0.275	»	56	0.655	0.27
2024	56	56	0	0.65	0.725	»	0.2	0.275	»	56	0.655	0.27
2025	57	58	1	0.65	0.725	4	0.2	0.275	4	58	0.66	0.265
2026	57	58	»	0.65	0.725	»	0.2	0.275	»	58	0.66	0.265
2027	57	58	»	0.65	0.725	»	0.2	0.275	»	58	0.66	0.265
2028	57	58	»	0.65	0.725	»	0.2	0.275	»	58	0.66	0.265
2029	57	58	»	0.65	0.725	»	0.2	0.275	»	58	0.66	0.265
2030	58	59	1	0.65	0.725	»	0.2	0.275	»	59	0.66	0.265
2031	58	59	»	0.65	0.725	»	0.2	0.275	»	59	0.66	0.265
2032	58	59	»	0.65	0.725	»	0.2	0.275	»	59	0.66	0.265
2033	58	59	»	0.65	0.725	»	0.2	0.275	»	59	0.66	0.265
2034	58	59	»	0.65	0.725	»	0.2	0.275	»	59	0.66	0.265
2035	59	60	1	0.65	0.725	4	0.2	0.275	4	59	0.69	0.275
2036	59	60	»	0.65	0.725	»	0.2	0.275	»	59	0.69	0.275
2037	59	60	»	0.65	0.725	»	0.2	0.275	»	59	0.69	0.275
2038	59	60	»	0.65	0.725	»	0.2	0.275	»	59	0.69	0.275
2039	59	60	»	0.65	0.725	»	0.2	0.275	»	59	0.69	0.275
2040	59	60	1	0.65	0.725	»	0.2	0.275	»	60	0.69	0.275
2041	60	61	»	0.65	0.725	»	0.2	0.275	»	60	0.69	0.275
2042	60	61	»	0.65	0.725	»	0.2	0.275	»	60	0.69	0.275
2043	60	61	»	0.65	0.725	»	0.2	0.275	»	60	0.69	0.275
2044	60	61	»	0.65	0.725	»	0.2	0.275	»	60	0.69	0.275
2045	61	62	1	0.65	0.725	4	0.2	0.275	4	61	0.67	0.275
2046	61	62	»	0.65	0.725	»	0.2	0.275	»	61	0.67	0.275
2047	61	62	»	0.65	0.725	»	0.2	0.275	»	61	0.67	0.275
2048	61	62	»	0.65	0.725	»	0.2	0.275	»	61	0.67	0.275
2049	61	62	»	0.65	0.725	»	0.2	0.275	»	61	0.67	0.275
2050	62	65	2	0.65	0.725	»	0.2	0.275	»	65	0.67	0.275
2051	62	65	»	0.65	0.725	»	0.2	0.275	»	65	0.67	0.275
2052	62	65	»	0.65	0.725	»	0.2	0.275	»	65	0.67	0.275
2053	62	65	»	0.65	0.725	»	0.2	0.275	»	65	0.67	0.275
2054	62	65	»	0.65	0.725	»	0.2	0.275	»	65	0.67	0.275
2055	65	66	1	0.65	0.725	4	0.2	0.275	4	66	0.66	0.275
2056	65	66	»	0.65	0.725	»	0.2	0.275	»	66	0.66	0.275
2057	65	66	»	0.65	0.725	»	0.2	0.275	»	66	0.66	0.275
2058	65	66	»	0.65	0.725	»	0.2	0.275	»	66	0.66	0.275
2059	65	66	»	0.65	0.725	»	0.2	0.275	»	66	0.66	0.275
2060	65	66	»	0.65	0.725	»	0.2	0.275	»	66	0.66	0.275

First of all, the objective function value is effectively zero for this case, implying a well balanced long term accumulated difference between pension expenditures and revenues, as seen in the related graph. Moreover, the results show that the minimum retirement ages in the solution increase gradually over the whole model horizon except for the sudden four year increase to be faced in 2050. The replacement rate shows a gradual and steady increase, from 0.65 to 0.69, approximately until the midpoint in the model horizon, 2030, followed by a similar decrease over the model horizon from 0.72 to 0.66, which is approximately the current value. The contribution rate also increases to an almost constant 50-year value of 0.275 after staying at 0.245 in the first ten years of the model horizon.

As can be seen from the yearly deficit graph, the minimum retirement age increase in 2050, combined with the 2045 decrease in RR brings the deficit level of SSI to the optimal value, approximately zero, in the long run. Figure 7 below shows the progression of accumulated deficit through the years.

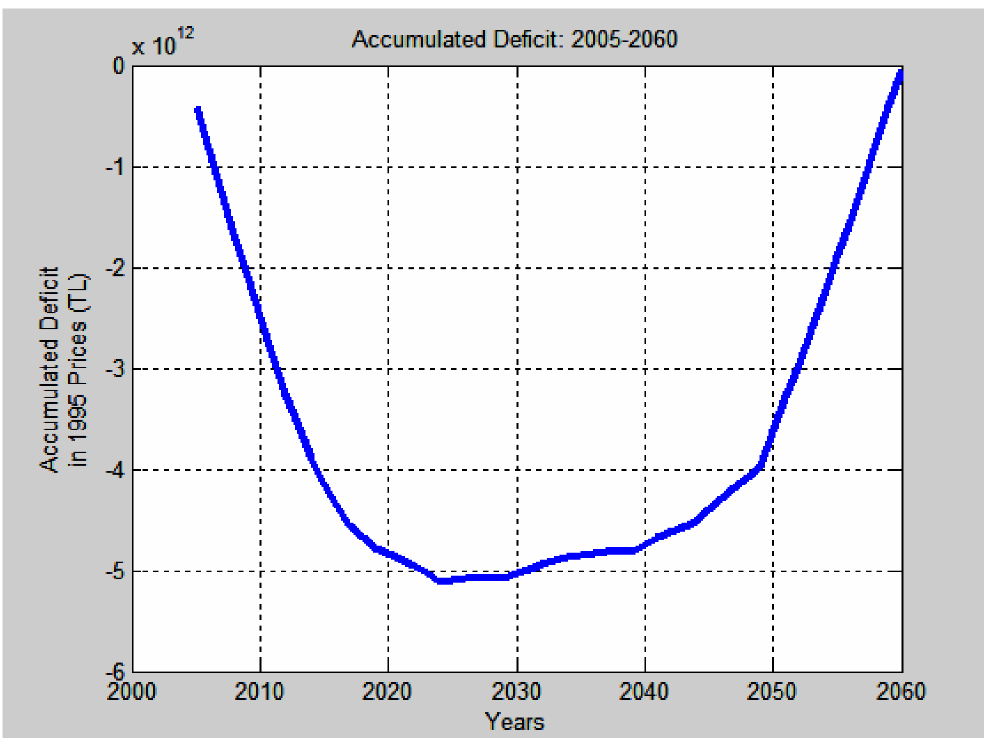

Figure 7: Accumulated Deficit for Scenario 1

An advantage of this scenario is its ability to keep replacement rates rather stable in the long run. Retirees registered to SSI almost always receive the same replacement rates without sudden drops.

A disadvantage of this proposal over the current proposal is its low RR over CR ratio in its final years. Scenario 1 has a RR over CR ratio of 2.4, whereas the current proposal's RR over CR ratio is 2.5.

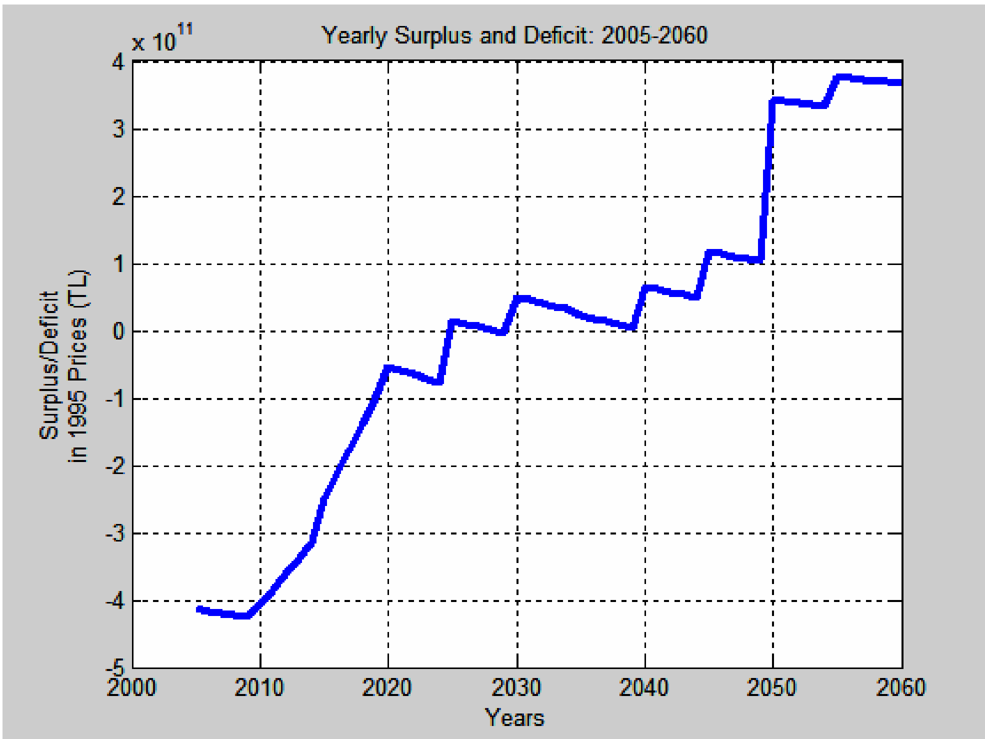

Figure 8: Yearly Surplus and Deficit for Scenario 1

Another disadvantage of this scenario is that it has very high surplus values at the end of the model horizon to balance the long run SSI deficits as shown above in Figure 8. However, these surpluses at the end of the model horizon will almost certainly continue to stay the same after the model horizon. Since the Turkish population was expected to reach its steady state after 2060, these surpluses will create large amounts of accumulated funds to be used elsewhere after 2060.

The improvement in the fitness value can be viewed in Figure 9 where the fitness shows a slow but steady increase in the first generations to face sudden improvement between generations 10 and 15. The fitness values are then almost constant in the last generations implying hinting a good solution.

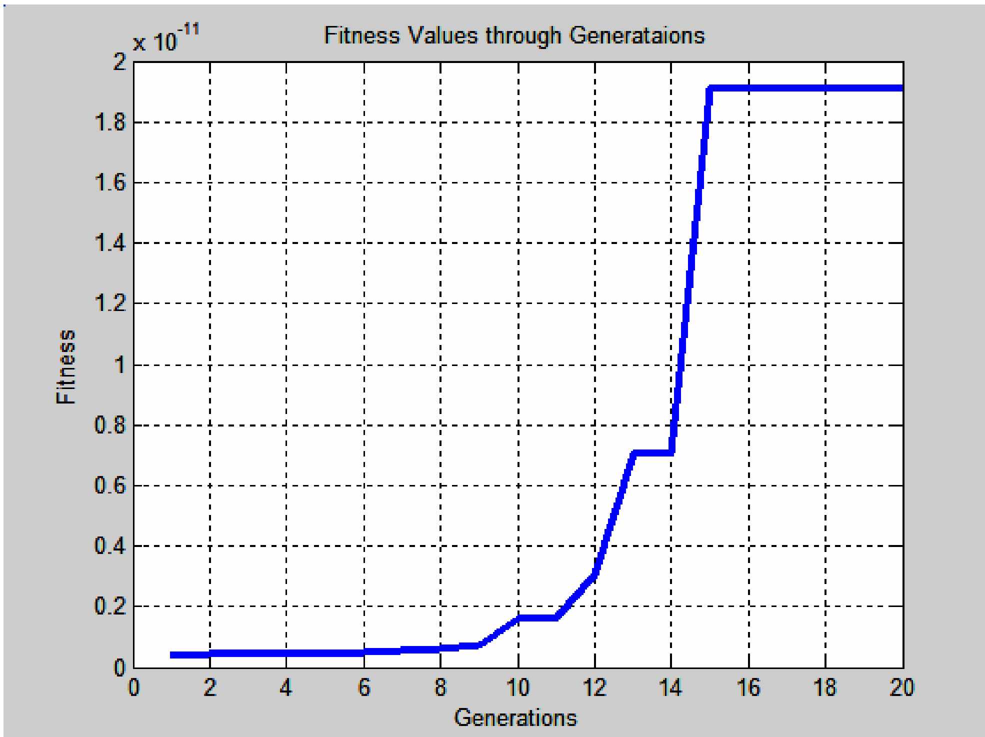

Figure 9: Fitness Values through Generations in Scenario 1

4.2 Scenario 2

In this exercise, the age structure of the current proposal will be conserved while allowing changes in RR and CR within the ranges of 0.65-0.725 and 0.2-0.275, respectively in years 2005, 2020, and 2040. 20 years is picked as a period of reasonable length with the intention of reflecting the effects of basic demographic changes in the workforce. The policy change years are projected to be between elections as in the previous scenario. The allowed ranges of pension parameters are presented below in Table 3.

Table 3: Allowed Ranges and the GA Output for Scenario 1

	Allowed Ranges of Pension Parameters									Results		
Year	min MRA	max MRA	Bit Depth	min. RR	max. RR	Bit Depth	CR	max. CR	Bit Depth	MRA	RR	CR
2005	41	41	0	0.65	0.725	4	0.2	0.275	4	41	0.675	0.255
2006	42	42	0	0.65	0.725	»	0.2	0.275	»	42	0.675	0.255
2007	43	43	0	0.65	0.725	»	0.2	0.275	»	43	0.675	0.255
2008	44	44	0	0.65	0.725	»	0.2	0.275	»	44	0.675	0.255
2009	45	45	0	0.65	0.725	»	0.2	0.275	»	45	0.675	0.255
2010	46	46	0	0.65	0.725	»	0.2	0.275	»	46	0.675	0.255
2011	47	47	0	0.65	0.725	»	0.2	0.275	»	47	0.675	0.255
2012	48	48	0	0.65	0.725	»	0.2	0.275	»	48	0.675	0.255
2013	49	49	0	0.65	0.725	»	0.2	0.275	»	49	0.675	0.255
2014	50	50	0	0.65	0.725	»	0.2	0.275	»	50	0.675	0.255
2015	51	51	0	0.65	0.725	»	0.2	0.275	»	51	0.675	0.255
2016	52	52	0	0.65	0.725	»	0.2	0.275	»	52	0.675	0.255
2017	53	53	0	0.65	0.725	»	0.2	0.275	»	53	0.675	0.255
2018	54	54	0	0.65	0.725	»	0.2	0.275	»	54	0.675	0.255
2019	55	55	0	0.65	0.725	»	0.2	0.275	»	55	0.675	0.255
2020	56	56	0	0.65	0.725	4	0.2	0.275	4	56	0.65	0.24
2021	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2022	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2023	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2024	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2025	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2026	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2027	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2028	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2029	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2030	58	58	0	0.65	0.725	»	0.2	0.265	»	58	0.65	0.24
2031	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2032	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2033	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2034	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2035	58	58	0	0.65	0.725	»	0.2	0.275	»	58	0.65	0.24
2036	59	59	0	0.65	0.725	»	0.2	0.275	»	59	0.65	0.24
2037	60	60	0	0.65	0.725	»	0.2	0.275	»	60	0.65	0.24
2038	60	60	0	0.65	0.725	»	0.2	0.275	»	60	0.65	0.24
2039	60	60	0	0.65	0.725	»	0.2	0.275	»	60	0.65	0.24
2040	61	61	0	0.65	0.725	4	0.2	0.265	4	61	0.695	0.27
2041	61	61	0	0.65	0.725	»	0.2	0.275	»	61	0.695	0.27
2042	62	62	0	0.65	0.725	»	0.2	0.275	»	62	0.695	0.27
2043	62	62	0	0.65	0.725	»	0.2	0.275	»	62	0.695	0.27
2044	63	63	0	0.65	0.725	»	0.2	0.275	»	63	0.695	0.27
2045	63	63	0	0.65	0.725	»	0.2	0.275	»	63	0.695	0.27
2046	64	64	0	0.65	0.725	»	0.2	0.275	»	64	0.695	0.27
2047	64	64	0	0.65	0.725	»	0.2	0.275	»	64	0.695	0.27
2048	65	65	0	0.65	0.725	»	0.2	0.275	»	65	0.695	0.27
2049	65	65	0	0.65	0.725	»	0.2	0.275	»	65	0.695	0.27
2050	65	65	0	0.65	0.725	»	0.2	0.275	»	65	0.695	0.27
2051	65	65	0	0.65	0.725	»	0.2	0.275	»	65	0.695	0.27
2052	65	65	0	0.65	0.725	»	0.2	0.275	»	65	0.695	0.27
2053	65	65	0	0.65	0.725	»	0.2	0.275	»	65	0.695	0.27

Table 3: Con't

2054	65	65	0	0.65	0.725	»	0.2	0.275	»	65	0.695	0.27
2055	66	66	0	0.65	0.725	»	0.2	0.275	»	66	0.695	0.27
2056	66	66	0	0.65	0.725	»	0.2	0.275	»	66	0.695	0.27
2057	66	66	0	0.65	0.725	»	0.2	0.275	»	66	0.695	0.27
2058	66	66	0	0.65	0.725	»	0.2	0.275	»	66	0.695	0.27
2059	66	66	0	0.65	0.725	»	0.2	0.275	»	66	0.695	0.27
2060	66	66	0	0.65	0.725	»	0.2	0.275	»	66	0.695	0.27

Once again, the accumulated deficit is very close to zero as shown in Figure 10 below and the result is optimal in this sense.

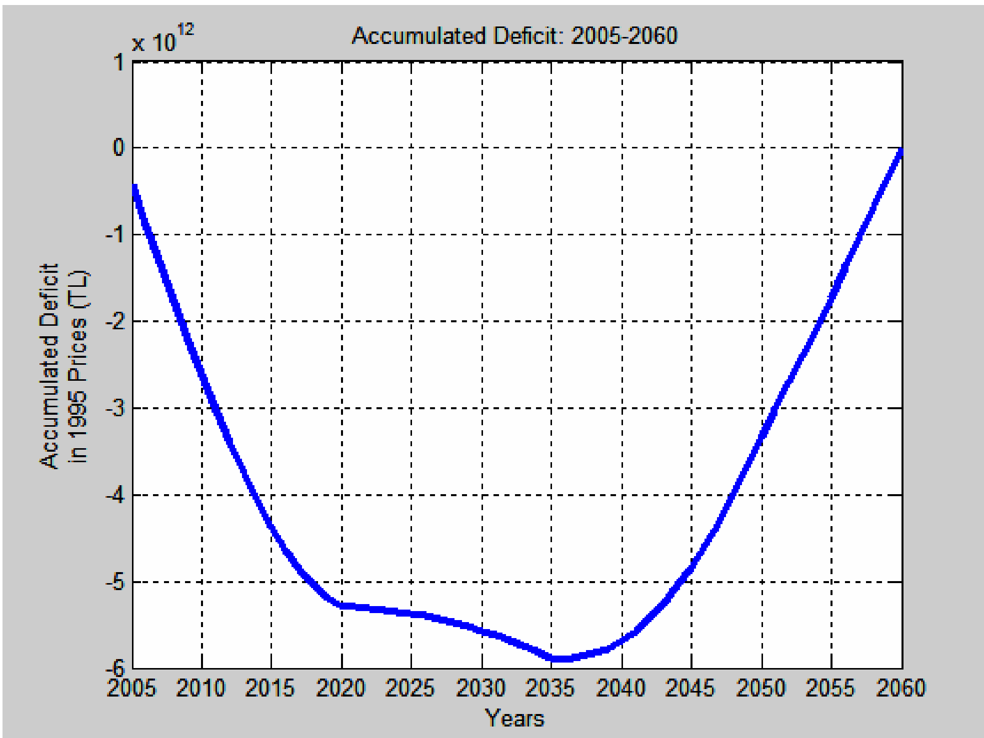

Figure 10: Accumulated Deficit for Scenario 2

One is tempted to interpret the results in this exercise as a balancing act if one especially notes that the values of RR and CR over the years show a positive correlation. In the 2005-2020 period RR and CR figures are respectively 0.675 and 0.255, whereas in the following period, 2021-2040, both values decrease to 0.65 and 0.24. Furthermore, in the next period, the values of both parameters increase again staying a few points above their initial values. Initially, this comovement might seem as a yearly deficit balancing act, since it is reasonable to think that when RR (liabilities) decreases, CR (assets) should also decrease. However true, this is not the force that drives the system to balance. If it were, observing an increase in deficit for the 2005-2020 period with the RR-CR pair 0.675 and 0.255, should project the same increase in deficit for the 2005-2020 period with nearly the same RR-CR pair 0.695 and 0.27. Interestingly, the opposite is the case. The 0.695-0.27 pair provides the upward thrust in the balances for the next 20 years due to the resulting change in the worker-retiree composition over the years.

Figure 11 below shows the yearly progress of total number of workers and retirees across all ages in the original and policy induced worker/retiree matrices.

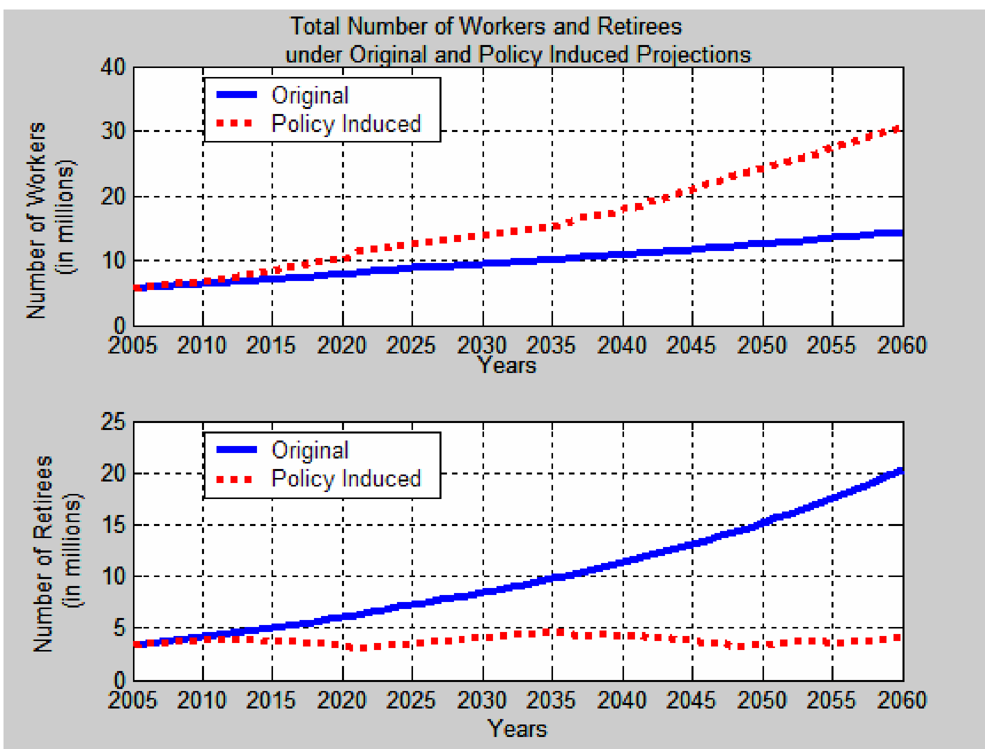

Figure 11: Original and Policy Induced Workforce Projections

The upward thrust in balances after 2040 is created by the change in the demographics of workforce. Therefore, in this solution, the 20-year periods reflect the general demographic trends in the sense that numerical increases in the workforce creates the fund necessary to sustain the previous periods' RR/CR ratios without necessarily decreasing this ratio drastically.

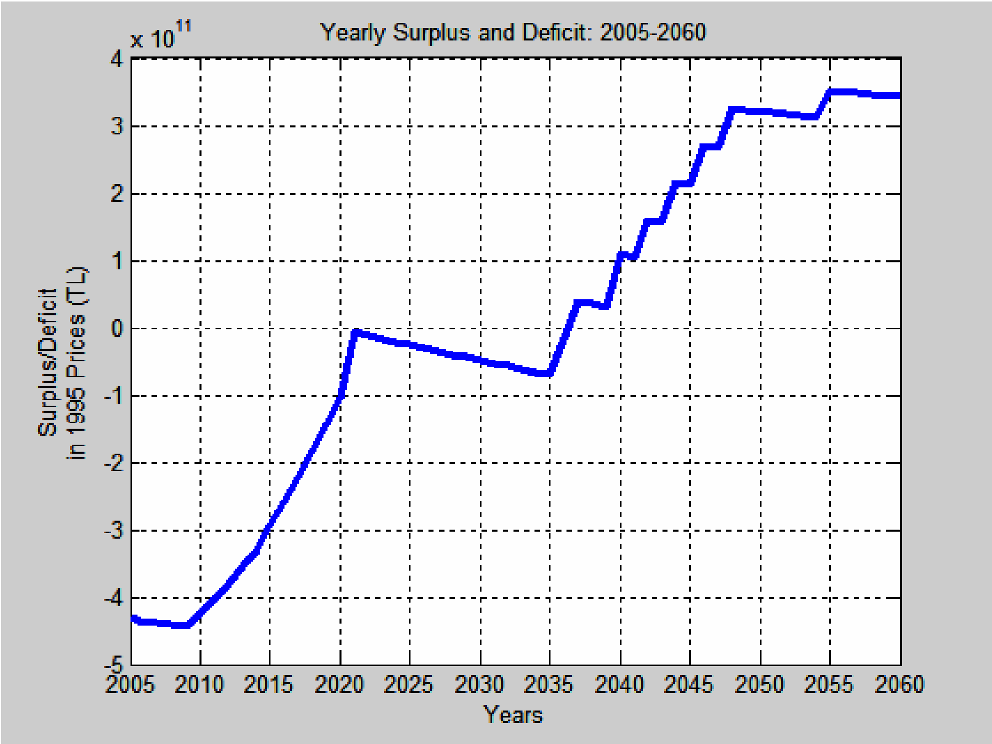

Figure 12: Yearly Surplus and Deficit for Scenario 2

Unfortunately, this scenario also has the same problem with the previous one in the sense that it has high surplus value at the end of the model horizon as shown in the above Figure 12. Moreover, another disadvantage of this scenario is its vulnerability to manipulation by the workers. The policy change in year 2040 provides incentive for many workers to retire in that year after paying low levels of contribution rates (0.24 instead of 0.27 after 2040) to receive higher levels of replacement rates (0.695 instead of 0.65 before 2040). Therefore, this scenario could cause significant changes in worker-retiree composition in the year 2040, should it be preannounced.

The progression of fitness values can be found in Figure 13 below.

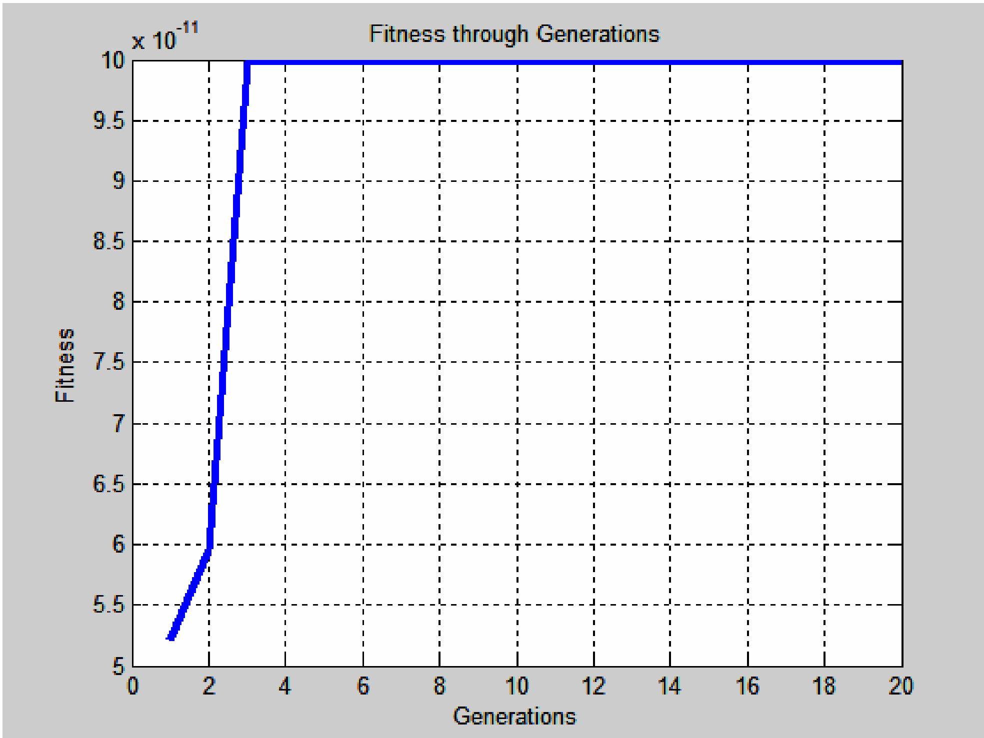

Figure 13: Fitness Values through Generations in Scenario 2

In this case the best fitness value found in the first few generations did not undergo any changes and stayed constant in the following generations.

4.3 Elasticity Sensitivity Analysis for Scenarios

Sensitivity analyses of results were made by considering alternative values of elasticity constants. For this purpose, the constants were increased by 50% and decreased by 50% to accommodate possible errors in elasticity measurements.

The result for Scenario 1 is presented in Figure 14.

The figure shows that the changes in elasticity values do not cause drastic changes in the accumulated deficit. The deficit increases as the elasticity constants increase due to the fact that higher values of elasticity constants would facilitate early retirement, leading to increasing expenditure and decreasing revenue for the SSI. The deficit decreases, on the other hand, led to the opposite result by partially restricting the early retirement process.

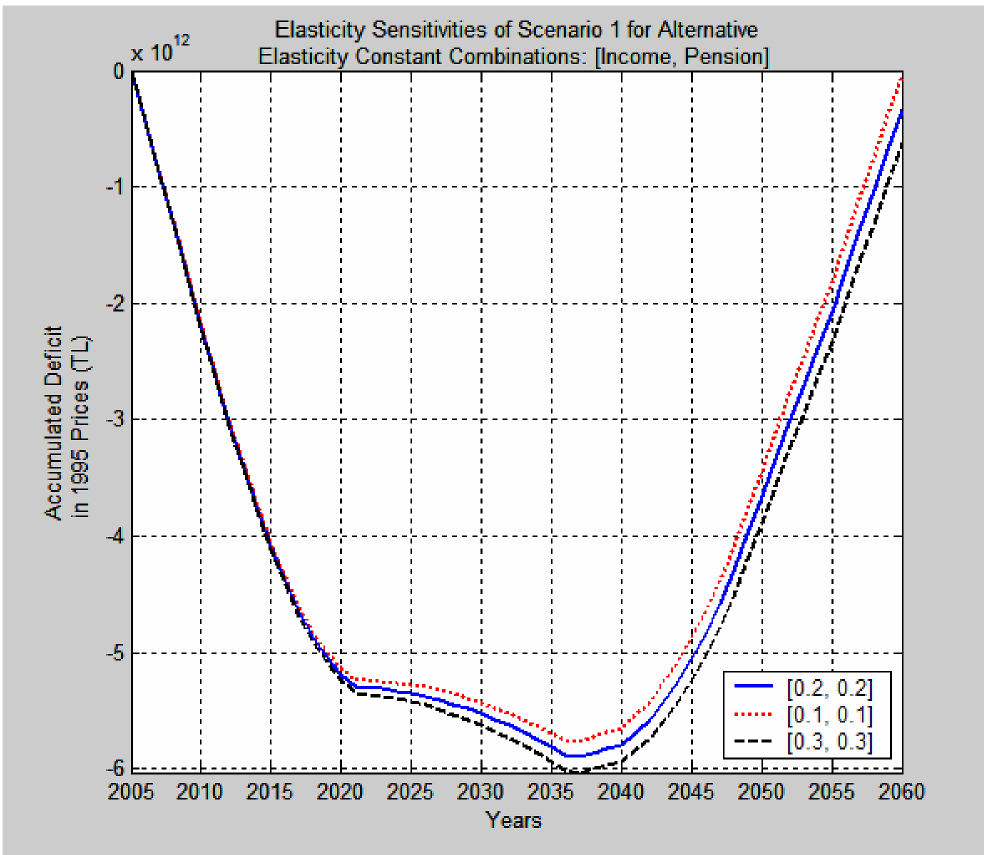

Figure 14: Elasticity Sensitivity Analysis for Scenario 1

The results for Scenario 2 is presented in Figure 15.

As in Scenario 1, the figure shows that the elasticity changes do not cause drastic changes in the accumulated deficit for Scenario 2. The deficit increases/decreases as the elasticity constants increase/decrease as discussed for Scenario 1.

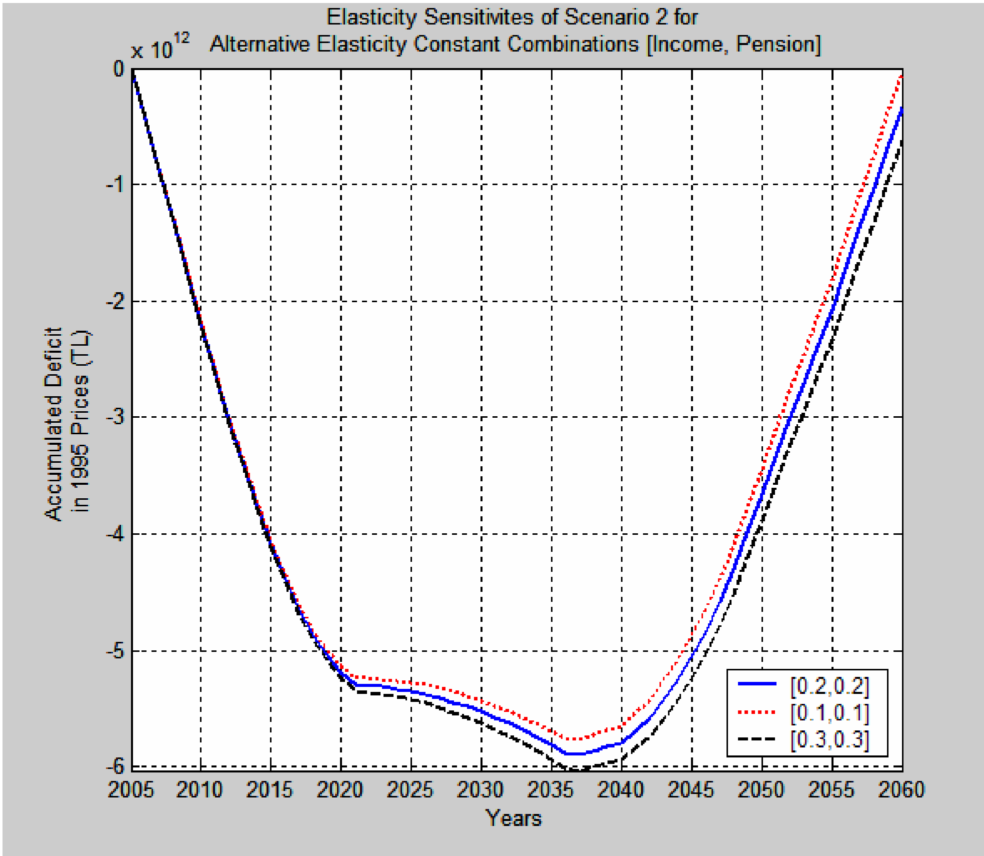

Figure 15: Elasticity Sensitivity Analysis for Scenario 2

Thus, it can be concluded that results from both scenarios are robust to changes in elasticities by as much as 50%.

CHAPTER 5

CONCLUSIONS

This thesis aimed to discuss a more realistic way of modeling parametric pension reform alternatives in Turkey by extending the Sayan and Turhan-Sayan (2001) article to model the effects of replacement and contribution rate changes with work and pension income elasticities of labor supply. Moreover, policy scenario options not available in the Sayan and Turhan-Sayan model, such as multiple time slots for changes in replacement and contribution rates were added to the GA program to provide greater flexibility to policy makers in designing a long term parametric reform.

The first scenario was designed to allow changes in the minimum retirement age in every five years, and changes in CR and RR every ten years. This scenario yielded a gradual and steady increase in minimum retirement age, combined with higher replacement rate values as well as higher contribution rates relative to the current proposal. As expected, the accumulated deficit turned out to be zero. However, this scenario produced large surpluses in the final years of the model horizon, and had a lower RR/CR ratio than the current one.

The second scenario was designed to create policy changes in every twenty years for CR and RR while keeping the MRA structure of the current proposal. The aim of considering twenty year intervals for policy changes was to adjust parameters along with changes in demographics over the model horizon. Again, as expected, the scenario had almost zero accumulated deficit. Furthermore, it had a higher RR/CR ratio than the current proposal. Yet, high surpluses in the final years remained. Under this scenario, the deficit turned out to improve mostly due to the significant changes in worker retiree populations.

Sensitivity analyses were made on accumulated deficits by changing the elasticity constants. A 50% change in both directions caused minimal changes in accumulated deficits, indicating that both scenarios were robust to ±50% changes in elasticities.

Further possible extensions to the model include modeling time-varying elasticity constants to consider effects of possible external shocks or behavioral changes due to varying economic circumstances.

The scenarios considered in this thesis make up only a small fraction of the possible scenarios that can be evaluated with the help of the GA developed here.

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APPENDIX

A.1 The Main GA Program (mainga)

```
% Genetic Algorithms Program for Social Security Forecast
%by M. Artun Alparslan

clear all;

%Clears variables in the workspace to avoid confusion and possible indexing
%errors.

load('original data wspace.mat');

%Loads original wage (wpay), average wage (wpayavg), retiree population
%(rat), and worker population (wat) matrices.

question=[2000 2002 38 38 1 0.65 0.65 0 0.20 0.20 0;
2003 2003 39 39 0 0.65 0.65 1 0.20 0.20 0;
2004 2004 40 40 0 0.65 0.65 2 0.20 0.20 0;
2005 2005 41 41 0 0.65 0.725 4 0.20 0.275 4;
2006 2006 42 42 0 0.65 0.725 inf 0.20 0.275 inf;
2007 2007 43 43 0 0.65 0.725 inf 0.20 0.275 inf;
2008 2008 44 44 0 0.65 0.725 inf 0.20 0.275 inf;
2009 2009 45 45 0 0.65 0.725 inf 0.20 0.275 inf;
2010 2010 46 46 0 0.65 0.725 inf 0.20 0.275 inf;
2011 2011 47 47 0 0.65 0.725 inf 0.20 0.275 inf;
2012 2012 48 48 0 0.65 0.725 inf 0.20 0.275 inf;
2013 2013 49 49 0 0.65 0.725 inf 0.20 0.275 inf;
2014 2014 50 50 0 0.65 0.725 inf 0.20 0.275 inf;
2015 2015 51 51 0 0.65 0.725 inf 0.20 0.275 inf;
2016 2016 52 52 0 0.65 0.725 inf 0.20 0.275 inf;
2017 2017 53 53 0 0.65 0.725 inf 0.20 0.275 inf;
2018 2018 54 54 0 0.65 0.725 inf 0.20 0.275 inf;
2019 2019 55 55 0 0.65 0.725 inf 0.20 0.275 inf;
2020 2020 56 56 0 0.65 0.725 4 0.20 0.275 4;
2021 2021 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2022 2022 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2023 2023 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2024 2024 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2025 2025 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
```

```
2026 2026 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2027 2027 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2028 2028 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2029 2029 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2030 2030 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2031 2031 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2032 2032 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2033 2033 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2034 2034 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2035 2035 58 58 0 0.65 0.725 inf 0.20 0.275 inf;
2036 2036 59 59 0 0.65 0.725 inf 0.20 0.275 inf;
2037 2037 60 60 0 0.65 0.725 inf 0.20 0.275 inf;
2038 2038 60 60 0 0.65 0.725 inf 0.20 0.275 inf;
2039 2039 60 60 0 0.65 0.725 inf 0.20 0.275 inf;
2040 2040 61 61 0 0.65 0.725 4 0.20 0.275 4;
2041 2041 61 61 0 0.65 0.725 inf 0.20 0.275 inf;
2042 2042 62 62 0 0.65 0.725 inf 0.20 0.275 inf;
2043 2043 62 62 0 0.65 0.725 inf 0.20 0.275 inf;
2044 2044 63 63 0 0.65 0.725 inf 0.20 0.275 inf;
2045 2045 63 63 0 0.65 0.725 inf 0.20 0.275 inf;
2046 2046 64 64 0 0.65 0.725 inf 0.20 0.275 inf;
2047 2047 64 64 0 0.65 0.725 inf 0.20 0.275 inf;
2048 2048 65 65 0 0.65 0.725 inf 0.20 0.275 inf;
2049 2049 65 65 0 0.65 0.725 inf 0.20 0.275 inf;
2050 2050 65 65 0 0.65 0.725 inf 0.20 0.275 inf;
2051 2051 65 65 0 0.65 0.725 inf 0.20 0.275 inf;
2052 2052 65 65 0 0.65 0.725 inf 0.20 0.275 inf;
2053 2053 65 65 0 0.65 0.725 inf 0.20 0.275 inf;
2054 2054 65 65 0 0.65 0.725 inf 0.20 0.275 inf;
2055 2055 66 66 0 0.65 0.725 inf 0.20 0.275 inf;
2056 2056 66 66 0 0.65 0.725 inf 0.20 0.275 inf;
2057 2057 66 66 0 0.65 0.725 inf 0.20 0.275 inf;
2058 2058 66 66 0 0.65 0.725 inf 0.20 0.275 inf;
2059 2059 66 66 0 0.65 0.725 inf 0.20 0.275 inf;
2060 2060 66 66 0 0.65 0.725 inf 0.20 0.275 inf;]

%Optimization problem to be solved is entered to the program through a
%matrix named "question". "question" is an n by 11 matrix where n is the
%number of time-slots. A single timeslot question vector consists of the
%following elements.

%question=[initial year, final year, min retirement age, max retirement
%age, retirement-bit-depth min RR, max RR, RR-bit-depth, min CR, max CR,
%CR-bit-depth ]

%Initial Year specifies the initial year of the time slot discussed.

%Final Year specifies the final year of the time slot discussed and it
%should always be one less than the next slot's "initial year" unless the
%time slot is the last time slot.
```

```
%Min Retirement Age, Min RR, and Min CR show the minimum values the
%corresponding variables can take in the given time slot.

%Max Retirement Age, Max RR, and Max CR show the maximum values the
%corresponding variables can take in the given time slot.

%Retirement Age Bit-Depth, RR Bit-Depth, and CR Bit-Depth show the
%bit-depth for variables in a given time slot. A bit depth of zero shows
%that the variable is constant and takes the value of the corresponding
%variable in its time-slot.  A bit depth of "inf" ("inf" was used here
%solely because of its nice properties in MATLAB such as being recognized
%by an if clause) implies that the variable is again constant but takes the
%value of previous time-slot. This helps in creating overlapping policies.

%A policy matrix is a 3 by 61 matrix where column carries the Retirement
%Age, RR, and CR information in its rows for all years in the horizon.


popsize = input('Population size: ');

%Enter population size


delta= input('Discounting Factor');

%Enter discounting factor

wage_elasticity=0.2;
pension_elasticity=0.2;
mutation_probability=0.07;

%Wage and Pension Elasticity constants along with mutation probability in
%a chromosome is chosen.

chromosome_size=0;

%Chromosome size for particular question ( for initiation)


index=[5,8,11];

%Indices of Bit-Depth variables (Retirement Age, RR, and CR respectively)to
%calculate chromosome size.

for i=1:size(question,1)
   for ind=1:3 %yas,rr,cr
      if question(i,index(ind))~=inf
         chromosome_size=chromosome_size+question(i,index(ind));
      end
```

```
    end
end
chromosome_size

%Every bit depth index in the given "question" matrix is traversed to sum
%their values to find the chromosome_size. "Inf"'s are treated as zeros
%naturally.

first_generation= ceil(rand(popsize, chromosome_size)-0.5);

%The first generation of pop_size chromosomes are created. Rand returns
%values between zero and one. Therefore, subtracting 0.5 and rounding up to
%the nearest greatest integer yields a chromosome made of zeros and ones.


for turns=1:20

    %Number of Turns is set to twenty.

    for pop=1:popsize

        %For all chromosomes in the population...
            whereAmI=1;

            %Sets a place marker at the first bit of the chromosome.

        for i=1:size(question,1)

            %and for all time slots in the given chromosome...

            for ind=1:3 %yas,rr,cr

            % Bit-Depths of Retirement Age, RR, and CR are transversed...

                if question(i,index(ind))==0
                    for j=question(i,1):question(i,2)
                        policy(ind,j-1999)=question(i,index(ind)-2);
                    end

                    %if the index is found to be zero the variable is
                    %constant, thus copied to the its corresponding place
                    %in the policy matrix.

                elseif question(i,index(ind))==inf
                    for j=question(i,1):question(i,2)
                        policy(ind,j-1999)=policy(ind,j-2000);
                    end

                    %if the index is found to be "inf" the variable is
                    %the same as the variable in the previous time slot.
```

```
            %That value is copied to its corresponding place
            %in the policy matrix.

        else

            %if the index is found to be a counting number,
            %then it means that we have encountered a part of the
            %chromosome. We evaluate the policy value induced by
            %this part of the chromosome in the policy by...

            for k=1:question(i,index(ind))
                powerarray(k)=2^(k-1);
            end

            %Learning its bit-depth from the current question
            %matrix and then calculating the powers of 2 for the
            %length of this bit-depth.

            for j=question(i,1):question(i,2)
                policy(ind,j-1999)=first_generation(pop,whereAmI:
                whereAmI-1+question(i,index(ind)))*powerarray'*
                ((question(i,index(ind)-1)-question(i,index(ind)-2)
                )/(2^question(i,index(ind))-1))+question
                (i,index(ind)-2);
            end

            %For each year in the time slot, policy variable is
            %calculated using the powers of two and the
            %corresponding bit values in the chromosome. The basic
            %idea is adding the minimum policy value (question
            %(i,index(ind)-2)) to the calculated step size
            %((question(i,index(ind)-1)-question(i,index(ind)-2))/
            %(2^question(i,index(ind))-1)) times the modulo 2 value
            %of the chosen part of the chromosome.

            whereAmI=whereAmI+question(i,index(ind));

            %Place marker is set to the beginning the next part of
            %the chromosome to be evaluated

            powerarray=0;

            %Powers of two function is nulled until the next call.

        end
    end
end

%Thus any arbitrary chromosome is changed into a policy
```

```
    newwatrat;

    %Each policy creates its own wat-rat table for evaluating its
    %fitness.

    deficit=0;
    for i=6:(question(size(question,1),2)-1999)
        deficit=(1/(1+delta)^(i-1-5))*(policy(3,i)*wpay(:,i)'
        *watupdated(:,i)-policy(2,i)*wpayavg(:,i)'*ratupdated(:,i))
        + deficit;
    end

    %OBJECTIVE FUNCTION is calculated...
    %For all years in the horizon, 2005-2060 (the index i starts from
    %6 since the dataset contained 2000-2060 data), the revenues of
    %SSI CR*wages*watupdated for all ages available is calculated and
    %subtracted from the expenditures of SSI, CR*avgwages*ratupdated
    %for all ages available, to be brought to 2005 terms by the
    %discounting function. The deficit for each year is added to the
    %total deficit to find the accumulated deficit.

    fitness=abs(1/(deficit+0.0001));

    % Fitness is found by reciprocating the total deficit.

    first_generation(pop,chromosome_size+1)=fitness;

    %Corresponding fitness values is appended to the parent
    %chromosomes.

end

fitSum=sum(first_generation(:,chromosome_size+1));

%Fitnesses of all chromosomes are added.

[topfitsinturns(turns), top_fit_row]=
max(first_generation(:,chromosome_size+1));
second_generation(1,:)=
first_generation(top_fit_row,1:chromosome_size);

%The fittest chromosome is transferred to the next generation
%directly (Elitism)

fit(turns,:)=first_generation(top_fit_row,:);

%The fittest elemnt in the generation is saved for archiving purposes.

for pop=2:popsize
```

```
%Creation of next pop_size-1 children.

for i=1:2
   n=rand*fitSum;

   %select a random point in the fit sum.

   head_count=0;
   okflag=0;
   current_sum=0;
   while okflag==0

      %We move on to find the parent by adding the fitnesses the
      %chromosomes until
      %we first exceed the random fitness point. That point
      %becomes our parent.

      if current_sum+
         first_generation(head_count+1,chromosome_size+1)>n
         okflag=1;
         parent(i,:)=
         first_generation(head_count+1,1:chromosome_size);

         %if the random point is less than the sum of variables,
         %the chromosome with the next head count value is
         %chosen as the parent.

      else
         current_sum=current_sum+
         first_generation(head_count+1,chromosome_size+1);
         head_count=head_count+1;

       %Add the fitness of the current chromosome if the current
       %fitness sum value is still less than the random one. It
       %means that we still could not find the chromosome we
       %were searching for.

      end
   end

   %We do this operation for two parents

end
cross_over_point=ceil(rand*chromosome_size);
chosen_child=ceil(rand+0.5);

%Two parents are found. Cross-over points are chosen. The child to
%be chosen is decided by another random operator.

temp=parent(1,cross_over_point);
```

```
        parent(1,cross_over_point)=parent(2,cross_over_point);
        parent(2,cross_over_point)=temp;

        %Cross-over at the chosen point is made. Two children are
        % created.
        if rand<mutation_probability
            mutation_point=ceil(rand*chromosome_size);
            parent(chosen_child,mutation_point)=
            abs(floor(parent(chosen_child,mutation_point)-0.5));
        end

        %Mutation at an arbitrary point is made with 0.07 probability.

        second_generation(pop,:)=parent(chosen_child,:);

        %Chosen child becomes member of the next generations.

    end
    first_generation=second_generation;

    %Generation of children become parents.

end

%The algorithms creates the populations for all turns and then finds out
%the best chromosomes. The best chromosome of the final population becomes
%the policy choice once it is converted to a policy by the below code,
%which was already explained above.


whereAmI=1;
        for i=1:size(question,1)
            for ind=1:3
                if question(i,index(ind))==0
                    for j=question(i,1):question(i,2)
                        policy(ind,j-1999)=question(i,index(ind)-2);
                    end
                elseif question(i,index(ind))==inf
                    for j=question(i,1):question(i,2)
                        policy(ind,j-1999)=policy(ind,j-2000);
                    end
                else
                    for k=1:question(i,index(ind))
                        powerarray(k)=2^(k-1);
                    end
                    for j=question(i,1):question(i,2)
                        policy(ind,j-1999)=first_generation(pop,whereAmI:
                        whereAmI-1+question(i,index(ind)))*powerarray'
                        *((question(i,index(ind)-1)
                        -question(i,index(ind)-2))/
```

```
            (2^question(i,index(ind))-1))
            +question(i,index(ind)-2);
          end
          whereAmI=whereAmI+question(i,index(ind));
          powerarray=0;

        end
      end
    end
    %arbtirary chromosome changed into policy
    %code for calculating usefulness
    newwatrat;
    deficit(1:5)=0;
    deficityear(1:5)=0;
    for i=6:(question(size(question,1),2)-1999)
      deficit(i)=(1/(1+delta)^(i-1-5))*(policy(3,i)
      *wpay(:,i)'*watupdated(:,i)-policy(2,i)
      *wpayavg(:,i)'*ratupdated(:,i)) + deficit(i-1);
    end
```

A.2 The Population Update Program (newwatrat)

```
watupdated=wat;
ratupdated=rat;

% New wat and rat variables are created in the workspace.

for j=1:size(policy(3,:),2)

%Under the original matrix size(policy(3,:),2)=61, we are operating on all years

    for ages 38:(policy(1,j)-1)

    %We operate on ages 38 through minimum retirement age for the current
%year.

    watupdated(age-14,j)=watupdated(age-14,j)+ ratupdated(age-37,j);

%Retirees are transferred to be workers if they are not eligible for retirement.

ratupdated(age-37,j)=0;

%Number of such retirees naturally drop to zero.

  end
end
```

```
%Retirement Age Update is Complete


policyrrcr=[[0.65,0.2];policy(2:3,:)'];

%Pre-2000 situation is appended to policy vector for computational ease.

for j=1:61
   deltaCR=policyrrcr(j+1,2)-policyrrcr(j,2);
   CRold=policyrrcr(j,2);
   deltaRR=policyrrcr(j+1,1)-policyrrcr(j,1);
   RRold=policyrrcr(j,1);

%Policy difference from the previuos year is calculated.

   for year=j:61

   %For all years following the policy change year…

      for age=policy(1,j):75

      % for every cohort eligible for retirement…

         workerchange=-1/2*(wage_elasticity*watupdated(age-
14,year)*deltaCR/(1-CRold)+pension_elasticity*ratupdated(age-
37,year)*deltaRR/RRold);

          %the change in the number of workers is calculated.

         if workerchange<0
            watupdated(age-14,year)=watupdated(age-14,year)+workerchange;
            ratupdated(age-37,year)=ratupdated(age-37,year)-workerchange;

         %if this change is negative for any cohort, then by the Dwat function, this
affect is applied to that cohort.

         end
      end
   end
end


%The program finishes updating the worker-retiree tables. It is ready for
%evaluation.
```